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Pythagorean 犹豫模糊熵及其多属性群决策方法 *

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摘 要:针对模糊信息下的群决策问题,提出了一种基于 Pythagorean 犹豫模糊熵的多属性群决策方法;给出了 Pythagorean 犹豫模糊熵的公理化定义及计算公式;为克服传统 Pythagorean 犹豫模糊集规范化方法导致原始决策信息流失的不足,完善了基于 Pythagorean 犹豫模糊环境下的最小公倍数扩充方法,方法能有效地保持原始决策信息;又以 Pythagorean 犹豫模糊熵作为决策信息差异程度的度量,给出属性权重完全未知或部分已知情况下权重的确定方法,并定义了基于最小公倍数的 Pythagorean 犹豫模糊距离测度和 Pythagorean 犹豫模糊熵测度;构造了一种基于 Pythagorean 犹豫模糊熵的 TOPSIS 方法,并通过精准扶贫补贴项目案例说明了方法的可行性和有效性.

关键词:多属性群决策;Pythagorean 犹豫模糊熵;属性权重;最小公倍数扩充方法;TOPSIS
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0 引 言

自 Zadeh 首次提出模糊集[1]以来,人们对其进行了广泛而深入的研究,并提出了一些新的模糊集,例如直觉模糊集[2]、区间直觉模糊集[3]、二型模糊集[3]等研究不仅丰富和发展了模糊集的理论基础,而且极大地推广了模糊集的应用范围,使之能更多地应用于解决实际问题当中。在此基础上,Yager[4-5]提出了 Pythagorean 模糊集,Torra[6]提出了犹豫模糊集,这两者均有着各自突出的优点和应用前景,文献[7]将两者结合给出了一种新的模糊集:Pythagorean 犹豫模糊集。将此二者相结合不仅扩充了模糊集的理论基础,而且对实际问题中决策方案的选取也有非常重要的意义。

熵测度作为研究模糊集的一种重要工具,Zadeh[8]首次引入之后,Luca 和 Termini[9]又基于 Shannon 函数给出了模糊集中熵的公理化定义。Bruillo[10]提出了直觉模糊环境下熵的公理化定义,并给出了计算公式;随后,Szmidt 和 Kacprzyk[11]在文献[9]的基础上,将信息熵定义扩展到了直觉模糊环境当中。为解决直觉指数不稳定的问题,王和

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雷[12]又给出了一种新的直觉模糊熵的构造方法。在直觉模糊熵方面,文献[14]和文献[15]也做出了很多贡献。使得直觉模糊熵公理化定义更加完善。

在犹豫模糊环境中,Xu 和 Xia[15]为衡量犹豫模糊值与其补集的差异化程度,提出了犹豫模糊熵的公理化定义,并给出其计算公式;之后,文献[16]从犹豫模糊集中各元素与 0.5 的距离不同的角度出发,提出了一种新的公理化定义和计算公式;文献[17]基于得分函数和离差函数,设计了一种新的犹豫模糊熵的公理化定义。但是,目前关于 Pythagorean 犹豫模糊熵的研究较少。

提出一种基于 Pythagorean 犹豫模糊熵的多属性群决策方法。首先提出 Pythagorean 犹豫模糊熵的公理化定义,然后提出 Pythagorean 犹豫模糊上的计算公式,并提出一种基于 Pythagorean 犹豫模糊熵的多属性决策方法。最后,将新方法应用到精准扶贫项目决策中,验证其可行性和有效性。

1　基本概念

定义 1[1]　设 X 为论域,则 $A=\{<x,\mu_A(x)>|0\leqslant\mu_A(x)\leqslant1,x\in X\}$ 称为 X 上的一个模糊集,$\mu_A(x)$表示 X 中元素 $x\in A$ 的隶属度。

定义 2[2]　设 X 为论域,则

$$A=\{<x,\mu_A(x),v_A(x)>,0\leqslant\mu_A(x)+v_A(x)\leqslant1,x\in X\}$$

称为 X 上的一个直觉模糊集,其中

$$\mu_A(x):X\to[0,1],v_A(x):X\to[0,1]$$

分别表示 X 上元素 $x\in X$ 的隶属度和非隶属度

$$\pi_A(x)=1-\mu_A(x)-v_A(x)$$

为其犹豫度。

记 $\beta=(\mu_\beta,v_\beta)$ 为直觉模糊数,且满足条件 $0\leqslant\mu_\beta,v_\beta\leqslant1,0\leqslant\mu_\beta+v_\beta\leqslant1$。

定义 3[6]　设 X 为论域,

$$A=\{<x,h_A(x)>|x\in X\}$$

称为 X 上的一个犹豫模糊集,其中 $h_A(x)$表示 X 上元素 $x\in A$ 的所有可能隶属度组成的集合。

记 $h_A=\{h_1,h_2,\cdots,h_l\}$ 为犹豫模糊数,且满足条件 $0\leqslant h_i\leqslant1,i=1,2,\cdots,l,h_i\leqslant h_{i+1}$。

定义 4[4]　设 X 为论域,

$$A=\{<x,\mu_A(x),v_A(x)>|x\in X\}$$

称为一个 Pythagorean 模糊集,其中 $\mu_A(x):X\to[0,1],v_A(x):X\to[0,1]$分别表示元素 $x\in X$ 的隶属度和非隶属度,且对任意的 $x\in X$,有 $\mu_A^2(x)+v_A^2(x)\leqslant1$ 成立。

定义 5[7]　设 X 为论域,则称三元组

$$A=\{<x,M_A(x),N_A(x)>|x\in X\}$$

为 Pythagorean 犹豫模糊集(PHFS),其中 $M_A(x)$,$N_A(x)$均为[0,1]上的非空有限集合,分别表示元素 x 属于 A 所有可能的隶属度集合和所有可能的非隶属度集合,且对任意的 $x\in X,\mu_A(x)\in M_A(x),v_A(x)\in N_A(x)$,满足$(\mu_A^+(x))^2+(v_A^+(x))^2\leqslant1$。称

$$\Pi_A(x)=\bigcup_{\substack{\mu_A(x)\in M_A(x)\\ v_A(x)\in N_A(x)}}\left\{\pi_A(x)=\sqrt{1-\mu_A^2(x)-v_A^2(x)}\right\}$$

为 x 属于 A 所有可能的犹豫度的集合。

注 1　当对任意的 $x\in X,M_A(x)$和 $N_A(x)$均只有一个元素时,Pythagorean 犹豫模糊集就退化为 Pythagorean 模糊数。

注 2　称 $\alpha=<M_\alpha,N_\alpha>$为一个 Pythagorean 犹豫模糊数。记 PHFN 为所有 Pythagorean 犹豫模糊数构成的集合。

定义 6[18]　设

$$\alpha=<M_\alpha,N_\alpha>=<\{\mu_{\alpha1},\mu_{\alpha2},\cdots,\mu_{\alpha\#M_\alpha}\},\{v_{\alpha1},v_{\alpha2},\cdots,v_{\alpha\#N_\alpha}\}>$$

是一个 Pythagorean 犹豫模糊数,则称

$$\pi_\alpha=\sqrt{1-\left(\frac{1}{\#M_\alpha}\sum_{i=1}^{\#M_\alpha}\mu_{\alpha i}^2+\frac{1}{\#N_\alpha}\sum_{i=1}^{\#N_\alpha}v_{\alpha i}^2\right)}\qquad(1)$$

为 α 的犹豫度,其中$\#M_\alpha,\#N_\alpha$ 分别表示隶属度元素个数和非隶属度元素个数。

2　Pythagorean 犹豫模糊数的最小公倍数扩充法

由于在 Pythagorean 犹豫模糊决策环境下,决策者们对某些问题认知的差异,使得他们对于同一个方案可能会给予不同的 Pythagorean 犹豫模糊决策信息,可能会出现任意两个 Pythagorean 犹豫模糊数之间的隶属度集合和非隶属度集合长度不相等的情况。基于此,Liang 和 Xu[18]采用基于乐观—悲观准则的扩充方法对 Pythagorean 犹豫模糊数之间长

度不等的情况进行处理。

但文献[19]指出了文献[18]提出的方法的有效性是存在争议的,因为从现实角度出发,上述方案既没有解释其决策者决策的主观性,也没有对方法的有效性和必要性进行说明。同时,文献[20]为了规范化处理犹豫模糊语言术语集中的不同元素,提出了一种最小公倍数拓展原则,文献[21]采用最小公倍数拓展法对 Pythagorean 犹豫模糊集的隶属度(非隶属度)进行规范化处理,但文献[21]关于最小公倍数拓展法的说法较为粗略浅显,且没有给出详细的定义过程和算法步骤,所以,将文献[20]的扩充思想运用到 Pythagorean 犹豫模糊数的规范化处理中。具体如下:

定义 7 设

$$\alpha=<M_\alpha,N_\alpha>\in PHFN,r,s\in N$$

则称集合

$$\alpha^{r,s}=<M_\alpha^r,N_\alpha^s>=\left[\left\{\underbrace{\mu_1,\cdots,\mu_1}_{r},\cdots,\underbrace{\mu_{\#M_\alpha},\cdots,\mu_{\#M_\alpha}}_{r}\right\},\left\{\underbrace{v_1,\cdots,v_1}_{s},\cdots,\underbrace{v_{\#N_\alpha},\cdots,v_{\#N_\alpha}}_{s}\right\}\right]$$

为 α 的 r-s 扩充集,且称这种对 α 的扩充方法为 r-s 扩充规则。其中,隶属度集合

$$M_\alpha^r=\left\{\overbrace{\mu_1,\cdots,\mu_1}^{r},\cdots,\overbrace{\mu_{\#M_\alpha},\cdots,\mu_{\#M_\alpha}}^{r}\right\}$$

中的各个元素分别被重复扩充了 r 次,非隶属度集合

$$N_\alpha^s=\left\{\overbrace{v_1,\cdots,v_1}^{s},\cdots,\overbrace{v_{\# N_\alpha},\cdots,v_{\# N_\alpha}}^{s}\right\}$$

中的各个元素分别被重复扩充了 s 次。显然扩充后的 $\alpha^{r,s}$ 仍为 Pythagorean 犹豫模糊数。

例 1 设 Pythagorean 犹豫模糊数

$$\alpha_1=<\{0.1,0.2\},\{0.7,0.8,0.9\}>$$

则 α 的 2-2 扩充集为

$$\alpha_1^{2,2}=<\{0.1,0.1,0.2,0.2\},\{0.7,0.7,0.8,0.8,0.9,0.9\}>$$

定义 8 设

$$\alpha=<M_\alpha,N_\alpha>,\beta=<M_\beta,N_\beta>$$

是两个 Pythagorean 犹豫模数,$lcm(\#M_\alpha,\#M_\beta)$ 为 $\#M_\alpha$ 和 $\#M_\beta$ 的最小公倍数,$\mathrm{lcm}(\#N_\alpha,\#N_\beta)$ 为 $\#N_\alpha$ 和 $\#N_\beta$ 的最小公倍数,其中 $\#M_\alpha$,$\#N_\alpha$ 分别表示 α 的隶属度元素个数和非隶属度元素个数,$\#M_\beta$,$\#N_\beta$ 分别表示 β 的隶属度元素个数和非隶属度元素个数。令

$$\bar{\alpha}=\left[\left\{\underbrace{\mu_1,\cdots,\mu_1}_{\frac{\mathrm{lcm}(\#M_\alpha,\#M_\beta)}{\#M_\alpha}},\cdots,\underbrace{\mu_{\#M_\alpha},\cdots,\mu_{\#M_\alpha}}_{\frac{\mathrm{lcm}(\#M_\alpha,\#M_\beta)}{\#M_\alpha}}\right\},\left\{\underbrace{v_1,\cdots,v_1}_{\frac{\mathrm{lcm}(\#N_\alpha,\#N_\beta)}{\#N_\alpha}},\cdots,\underbrace{v_{\#N_\alpha},\cdots,v_{\#N_\alpha}}_{\frac{\mathrm{lcm}(\#N_\alpha,\#N_\beta)}{\#N_\alpha}}\right\}\right] \tag{2}$$

$$\bar{\beta}=\left[\left\{\underbrace{\mu_1,\cdots,\mu_1}_{\frac{\mathrm{lcm}(\#M_\alpha,\#M_\beta)}{\#M_\beta}},\cdots,\underbrace{\mu_{\#M_\alpha},\cdots,\mu_{\#M_\alpha}}_{\frac{\mathrm{lcm}(\#M_\alpha,\#M_\beta)}{\#M_\beta}}\right\},\left\{\underbrace{v_1,\cdots,v_1}_{\frac{\mathrm{lcm}(\#N_\alpha,\#N_\beta)}{\#N_\beta}},\cdots,\underbrace{v_{\#N_\alpha},\cdots,v_{\#N_\alpha}}_{\frac{\mathrm{lcm}(\#N_\alpha,\#N_\beta)}{\#N_\beta}}\right\}\right] \tag{3}$$

则称 $\bar{\alpha}$,$\bar{\beta}$ 分别为 α,β 的规范化的 Pythagorean 犹豫模糊数。

根据定义 7 和定义 8,任意 Pythagorean 犹豫模糊数均可以通过扩充方法,变成新的 Pythagorean 犹豫模糊数,扩充方法被称作最小公倍数扩充法。

若采用定义 8 对 Pythagorean 犹豫模糊数的隶属度和非隶属度进行扩充,则可得到基于最小公倍数扩充法的 Pythagorean 犹豫模糊数规范化方法,基于扩充 Pythagorean 犹豫模糊数的规范化算法如下:

输入:任意两个 Pythagorean 犹豫模糊数 $\alpha=<M_\alpha,N_\alpha>$,$\beta=<M_\beta,N_\beta>$。

输出:标准 Pythagorean 犹豫模糊数 $\bar{\alpha}=<M_{\bar{\alpha}},N_{\bar{\alpha}}>$,$\bar{\beta}=<M_{\bar{\beta}},N_{\bar{\beta}}>$。

步骤 1 计算 Pythagorean 犹豫模糊数 α 和 β 的元素个数,即:$\#M_\alpha,\#M_\beta,\#N_\alpha,\#N_\beta$。

步骤 2 计算 $\#M_\alpha$ 和 $\#M_\beta$ 的最小公倍数 $\mathrm{lcm}(\#M_\alpha,\#M_\beta)$,$\#N_\alpha$ 和 $\#N_\beta$ 的最小公倍数 $\mathrm{lcm}(\#N_\alpha,\#N_\beta)$。

步骤 3 计算 $\frac{\mathrm{lcm}(\#M_\alpha,\#M_\beta)}{\#M_\alpha}$,$\frac{\mathrm{lcm}(\#M_\alpha,\#M_\beta)}{\#M_\beta}$,$\frac{\mathrm{lcm}(\#N_\alpha,\#N_\beta)}{\#N_\alpha}$ 和 $\frac{\mathrm{lcm}(\#N_\alpha,\#N_\beta)}{\#N_\beta}$。

步骤 4 根据式(2),式(3)计算 $\alpha^{\frac{\mathrm{lcm}(\#M_\alpha,\#M_\beta)}{\#M_\alpha},\frac{\mathrm{lcm}(\#N_\alpha,\#N_\beta)}{\#N_\alpha}}$ 和 $\beta^{\frac{\mathrm{lcm}(\#M_\alpha,\#M_\beta)}{\#M_\beta},\frac{\mathrm{lcm}(\#N_\alpha,\#N_\beta)}{\#N_\beta}}$。

步骤 5 输出 $\bar{\alpha}=\alpha^{\frac{\mathrm{lcm}(\#M_\alpha,\#M_\beta)}{\#M_\alpha},\frac{\mathrm{lcm}(\#N_\alpha,\#N_\beta)}{\#N_\alpha}}$ 和

$\bar{\beta}=\beta^{\frac{\mathrm{lcm}(\#M_\alpha,\#M_\beta)}{\#M_\beta},\frac{\mathrm{lcm}(\#N_\alpha,\#N_\beta)}{\#N_\beta}}$。

步骤 6　结束。

例 2　设有两个 Pythagorean 犹豫模糊数

$$\alpha=<\{0.1,0.2\},\{0.7,0.8,0.9\}>$$

$$\beta=<\{0.7,0.8,0.9\},\{0.1,0.2\}>$$

根据算法 1,则有

$$\#M_\alpha=2,\#M_\beta=3,\#N_\alpha=3,\#N_\beta=2;$$

$$lcm(\#M_\alpha,\#M_\beta)=6,lcm(\#N_\alpha,\#N_\beta)=6;$$

得到标准 $\bar{\alpha}$,$\bar{\beta}$ 为

$$\bar{\alpha}=<\{0.1,0.1,0.1,0.2,0,2,0.2\},\{0.7,0.7,0.8,0.8,0.9,0.9\}>;$$

$$\bar{\beta}=<\{0.7,0.7,0.8,0.8,0.9,0.9\},\{0.1,0.1,0.1,0.2,0.2,0.2\}>。$$

定义 9[18]　设

$$\alpha=<\{\mu_1,\cdots,\mu_{\#M_\alpha}\},\{v_1,\cdots,v_{\#N_\alpha}\}>,$$

$$\beta=<\{\mu_1,\cdots,\mu_{\#M_\beta}\},\{v_1,\cdots,v_{\#N_\beta}\}>$$

是两个 Pythagorean 犹豫模糊数,则 α 和 β 之间的欧氏距离定义为

$$d(\alpha,\beta)=d(\alpha^{\frac{\tau}{\#M_\alpha},\frac{\varphi}{\#N_\alpha}},\beta^{\frac{\tau}{\#M_\beta},\frac{\varphi}{\#N_\beta}})=$$

$$\left(\frac{1}{2}\left(\frac{1}{\tau}\sum_{i=1}^{\tau}[(\mu_{\alpha i})^2-(\mu_{\beta i})^2]^2+\frac{1}{\varphi}\sum_{j=1}^{\varphi}[(v_{\alpha j})^2-(v_{\beta j})^2]^2+[(\pi_\alpha)^2-(\pi_\beta)^2]^2\right)\right)^{\frac{1}{2}}$$

其中,$\mathrm{lcm}(\#M_\alpha,\#M_\beta)=\tau$,$\mathrm{lcm}(\#N_\alpha,\#N_\beta)=\varphi$,$\pi_\alpha$ 和 π_β 由式(1)给出。

3　Pythagorean 犹豫模糊熵

Pythagorean 犹豫模糊熵可以有效度量模糊信息的模糊程度,能够将模糊程度较高的数据转化更有利于决策者精确评价的信息。对于两个 Pythagorean 犹豫模糊数:

$$\alpha=<\{\mu_1,\mu_2,\cdots,\mu_{\#M_\alpha}\},\{v_1,v_2,\cdots,v_{\#N_\alpha}\}>\in \mathrm{PHFN}$$

$$\beta=<\{\mu_1,\mu_2,\cdots,\mu_{\#M_\beta}\},\{v_1,v_2,\cdots,v_{\#N_\beta}\}>\in \mathrm{PHFN}$$

当 $\left|\frac{1}{\tau}\sum_{j=1}^{\tau}\mu_{\alpha j}^2-\frac{1}{\varphi}\sum_{j=1}^{\varphi}v_{\alpha j}^2\right|=\left|\frac{1}{\tau}\sum_{j=1}^{\tau}\mu_{\beta j}^2-\frac{1}{\varphi}\sum_{j=1}^{\varphi}v_{\beta j}^2\right|$ 时,若 $\pi(x)$ 越大则表明专家对 x 的未知程度越高,即模糊程度越高,其熵值应该越大,反之,若 $\pi(x)$ 越小则表明对 x 的未知程度越低,即模糊程度越低,其熵值应该越小;当 $\left|\frac{1}{\tau}\sum_{j=1}^{\tau}\mu_{\alpha j}^2-\frac{1}{\varphi}\sum_{j=1}^{\varphi}v_{\alpha j}^2\right|\neq\left|\frac{1}{\tau}\sum_{j=1}^{\tau}\mu_{\beta j}^2-\frac{1}{\varphi}\sum_{j=1}^{\varphi}v_{\beta j}^2\right|$ 时,$\left|\frac{1}{\tau}\sum_{j=1}^{\tau}\mu^2-\frac{1}{\varphi}\sum_{j=1}^{\varphi}v^2\right|$ 越小,即隶属度和非隶属度越接近时,表明决策者对 x 的不确定程度较高,其熵值应越大,反之,$\left|\frac{1}{\tau}\sum_{j=1}^{\tau}\mu^2-\frac{1}{\varphi}\sum_{j=1}^{\varphi}v^2\right|$ 越大,即两者差异性较大,表明决策者对 x 的不确定程度较低,其熵值应越小。为更加直观的判断 Pythagorean 犹豫模糊熵值的大小,同时体现隶属度、非隶属度及犹豫度对熵值的贡献,下面给出基于 Pythagorean 犹豫模糊环境下熵的公理化定义及计算公式:

定义 10　设 $A=\{\alpha_1,\alpha_2,\cdots,\alpha_n\}$ 是一个 Pythagorean 犹豫模糊集,其中

$$\alpha_i=<M_{\alpha_i},N_{\alpha_i}>=<\{\mu_{\alpha_i1},\mu_{\alpha_i2},\cdots,\mu_{\alpha_i\tau}\},\{v_{\alpha_i1},v_{\alpha_i2},\cdots,v_{\alpha_i\varphi}\}>$$

是一个 Pythagorean 犹豫模糊数,则 A 的熵可定义为

$$E(A)=\frac{1}{n}\sum_{i=1}^{n}\frac{1-\left|\frac{1}{\tau}\sum_{j=1}^{\tau}\mu_{\alpha_j}^2-\frac{1}{\varphi}\sum_{j=1}^{\varphi}v_{\alpha_j}^2\right|+\pi_\alpha^2(x)}{1+\left|\frac{1}{\tau}\sum_{j=1}^{\tau}\mu_{\alpha_j}^2-\frac{1}{\varphi}\sum_{j=1}^{\varphi}v_{\alpha_j}^2\right|+\pi_\alpha^2(x)}=$$

$$\frac{1}{n}\sum_{i=1}^{n}\frac{2-\left|\frac{1}{\tau}\sum_{j=1}^{\tau}\mu_{\alpha_j}^2-\frac{1}{\varphi}\sum_{j=1}^{\varphi}v_{\alpha_j}^2\right|-\frac{1}{\tau}\sum_{j=1}^{\tau}\mu_{\alpha_j}^2-\frac{1}{\varphi}\sum_{j=1}^{\varphi}v_{\alpha_j}^2}{2+\left|\frac{1}{\tau}\sum_{j=1}^{\tau}\mu_{\alpha_j}^2-\frac{1}{\varphi}\sum_{j=1}^{\varphi}v_{\alpha_j}^2\right|-\frac{1}{\tau}\sum_{j=1}^{\tau}\mu_{\alpha_j}^2-\frac{1}{\varphi}\sum_{j=1}^{\varphi}v_{\alpha_j}^2}\tag{3}$$

式(3)中,$\left|\frac{1}{\tau}\sum_{j=1}^{\tau}\mu^2-\frac{1}{\varphi}\sum_{j=1}^{\varphi}v^2\right|$ 表示隶属度与非隶属度之间的差异会影响到 Pythagorean 犹豫模糊集的不确定程度,同时,$\pi_\alpha^2(x)$ 凸显出犹豫度对 Pythagorean 犹豫模糊集未知性的贡献,因而,式(3)从不确定性和未知性方面合理且全面地刻画了 Pythagorean 犹豫模糊集的模糊度。

定理 1　设

$$\alpha=<M_\alpha,N_\alpha>=<\{\mu_{\alpha1},\mu_{\alpha2},\cdots,\mu_{\alpha\#M_\alpha}\},\{v_{\alpha1},v_{\alpha2},\cdots,v_{\alpha\#N_\alpha}\}>$$

$$\beta=<M_\beta,N_\beta>=<\{\mu_{\beta1},\mu_{\beta2},\cdots,\mu_{\beta\#M_\beta}\},\{v_{\beta1},v_{\beta2},\cdots,v_{\beta\#N_\beta}\}>$$

是两个 Pythagorean 犹豫模糊数, $E:\alpha,\beta\rightarrow[0,$

1]为 Pythagorean 犹豫模糊熵,则 E 满足以下性质:

(1) $E(\alpha)=0$,当且仅当

$$\alpha=<\{1,1,\cdots,1\},\{0,0,\cdots,0\}>$$

或

$$\alpha=<\{0,0,\cdots,0\},\{1,1,\cdots,1\}>$$

(2) $E(\alpha)=1$,当且仅当 $M_\alpha=N_\alpha$,即 $\sum_{i=1}^{\#M_\alpha}\mu_{\alpha i}^2=\sum_{i=1}^{\#N_\alpha}v_{\alpha i}^2$,且此时$\#M_\alpha=\#N_\alpha$;

(3) $E(\alpha)=E(\alpha^c)$,这里 $\alpha^c=<N_\alpha,M_\alpha>$;

(4) 当 $E(\alpha')\leqslant E(\beta')$时,有两种情况:

(Ⅰ) 当 $\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2\geqslant\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2$ 时,有 $\sum_{i=1}^{\tau}\mu_{\alpha'i}^2\geqslant\sum_{i=1}^{\tau}\mu_{\beta'i}^2,\sum_{i=1}^{\varphi}v_{\alpha'i}^2\leqslant\sum_{i=1}^{\varphi}v_{\beta'i}^2$ 成立;

(Ⅱ) 当 $\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2\leqslant\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2$ 时,有 $\sum_{i=1}^{\tau}\mu_{\alpha'i}^2\leqslant\sum_{i=1}^{\tau}\mu_{\beta'i}^2,\sum_{i=1}^{\varphi}v_{\alpha'i}^2\geqslant\sum_{i=1}^{\varphi}v_{\beta'i}^2$ 成立。

这里

$$\alpha'=<\{\mu_{\alpha'1},\mu_{\alpha'2},\cdots,\mu_{\alpha'\tau}\},\{v_{\alpha'1},v_{\alpha'2},\cdots,v_{\alpha'\varphi}\}>$$

$$\beta'=<\{\mu_{\beta'1},\mu_{\beta'2},\cdots,\mu_{\beta'\tau}\},\{v_{\beta'1},v_{\beta'2},\cdots,v_{\beta'\varphi}\}>$$

是 α,β 经最小公倍数扩充方法规范化处理后的 Pythagorean 犹豫模糊数。

证明

(1) ~ (3)显然成立,下证(4)。

(4)(⇒)显然成立;

(⇐)(Ⅰ)当 $\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2\geqslant\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2$ 时,有

$$\frac{2-\left|\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2\right|-\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2}{2+\left|\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2\right|-\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2}=\frac{1-\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2}{1-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2}$$

当 $\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\alpha'i}^2\geqslant\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2,\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\alpha'i}^2\leqslant\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2$ 时,有

$$\frac{1-\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2}{1-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2}\geqslant\frac{1-\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\alpha'i}^2}{1-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\alpha'i}^2}$$

故,$E(\alpha')\leqslant E(\beta')$。

(Ⅱ) 当 $\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2\leqslant\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2$ 时,有$\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2\leqslant 0$

$$\frac{2-\left|\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2\right|-\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2}{2+\left|\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2\right|-\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2}=\frac{1-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2}{1-\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2}$$

又 $\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\alpha'i}^2\leqslant\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2,\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\alpha'i}^2\geqslant\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2$,

$$\frac{1-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\beta'i}^2}{1-\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\beta'i}^2}\geqslant\frac{1-\frac{1}{\varphi}\sum_{i=1}^{\varphi}v_{\alpha'i}^2}{1-\frac{1}{\tau}\sum_{i=1}^{\tau}\mu_{\alpha'i}^2}$$

故 $E(\alpha')\leqslant E(\beta')$。证毕。

4 基于 Pythagorean 环境下的多属性群决策方法

考虑一个多属性群决策问题。设方案集为 $X=\{x_1,x_2,\cdots,x_m\}$,属性集 $C=\{c_1,c_2,\cdots,c_n\}$,决策者集为 $D=\{d_1,d_1,\cdots,d_t\}$,属性的权重向量为 $W=(w_1,w_2,\cdots,w_n)^T$,满足 $\sum_{j=1}^{n}w_j=1,0\leqslant w_j\leqslant 1,j=1,2,\cdots,n$,决策者的权重向量为 $L=(l_1,l_2,\cdots,l_t)^T$,且满足 $\sum_{k=1}^{t}l_k=1,0\leqslant l_k\leqslant 1,k=1,2,\cdots,t$。设 $R^{(k)}=(r_{ij}^{(k)})_{m\times n}(k=1,2,\cdots,t)$为 d_k 针对不同属性 c_j 在方案 x_i 下的评价结果,其中:$r_{ij}^{(k)}=<\{\mu_{ij_1}^{(k)},\mu_{ij_2}^{(k)},\cdots,\mu_{ij\#M_r}^{(k)}\},\{v_{ij_1}^{(k)},v_{ij_2}^{(k)},\cdots,v_{ij\#N_r}^{(k)}\}>$设 $E(\tilde{r}_{ij}^{(k)})$表示 $\tilde{r}_{ij}^{(k)}$ 的 Pythagorean 犹豫模糊熵,则属性 c_j 的熵可表示

$$E_j=\sum_{k=1}^{t}l_k\sum_{i=1}^{m}E(\tilde{r}_{ij}^{(k)}) \tag{4}$$

根据熵理论知,如果各方案针对同一属性下的熵值越小,说明该属性的稳定性越高,则应该被赋予一个较大的权重,即 E_j 越小,其权重 w_j 就越大。

若属性权重 w_j 完全未知,可由式(4)计算属性 c_j 的权重:

$$w_j = \frac{\sum_{k=1}^{t} l_k \sum_{i=1}^{m} (1 - E(\tilde{r}_{ij}^{(k)}))}{\sum_{j=1}^{n} \sum_{k=1}^{t} l_k \sum_{i=1}^{m} (1 - E(\tilde{r}_{ij}^{(k)}))},\quad j = 1,2,\cdots,n \tag{5}$$

若属性权重 w_j 部分已知，令 H 表示未知权重满足的条件集合，则可构建模型求解属性权重：

$$\min E_W = \sum_{k=1}^{t} l_k \sum_{i=1}^{m} \sum_{j=1}^{n} w_j E(r_{ij}^{(k)})$$
$$\text{s.t}\begin{cases} W \in H \\ \sum_{j=1}^{n} w_j = 1 \\ w_j \geqslant 0, j = 1,2,\cdots,n \end{cases} \tag{6}$$

TOPSIS 法是解决多属性问题的常用方法之一，其宗旨是找出最佳方案和最差方案，即为正理想点和负理想点。方法主要是通过计算离正理想点和负理想点的距离，之后再通过计算贴近度，对方案进行排序和择优。

在 Pythagorean 犹豫模糊环境下，正理想点为

$$r_{0j}^{(k)+} = <\{1,1,\cdots,1\},\{0,0,\cdots,0\}>$$

负理想点为

$$r_{0j}^{(k)-} = <\{0,0,\cdots,0\},\{1,1,\cdots,1\}>$$

则方案 x_i 与正理想点的加权距离为

$$AD_i^+ = \sum_{j=1}^{n} w_j \left(\frac{1}{2}\left(\frac{1}{\#M_r}\sum_{i=1}^{\#M_r}[(\mu_{ij}^{(k)})^2 - 1]^2 + \frac{1}{\#N_r}\sum_{i=1}^{\#N_r}(v_{ij}^{(k)})^4 + \left[1 - \frac{1}{\#M_r}\sum_{i=1}^{\#M_r}(\mu_{ij}^{(k)})^2 - \frac{1}{\#N_r}\sum_{i=1}^{\#N_r}(v_{ij}^{(k)})^2\right]^2\right)\right)^{\frac{1}{2}} \tag{7}$$

方案 x_i 与负理想点的加权距离为

$$AD_i^- = \sum_{j=1}^{n} w_j \left(\frac{1}{2}\left(\frac{1}{\#M_r}\sum_{i=1}^{\#M_r}(\mu_{ij}^{(k)})^4 + \frac{1}{\#N_r}\sum_{i=1}^{\#N_r}[(v_{ij}^{(k)})^2 - 1]^2 + \left[1 - \frac{1}{\#M_r}\sum_{i=1}^{\#M_r}(\mu_{ij}^{(k)})^2 - \frac{1}{\#N_r}\sum_{i=1}^{\#N_r}(v_{ij}^{(k)})^2\right]^2\right)\right)^{\frac{1}{2}} \tag{8}$$

其中 w_j 为属性 c_j 的权重。

方案 x_i 的贴近度可表示为

$$P_i = \frac{AD_i^-}{AD_i^+ + AD_i^-} \tag{9}$$

故基于 Pythagorean 犹豫模糊熵的多属性群决策步骤如下：

步骤 1　矩阵的规范化处理。设

$$\boldsymbol{R}^{(k)} = (\boldsymbol{r}_{ij}^{(k)})_{m\times n}, (k=1,2,\cdots,t)$$

为决策矩阵，其中：

$$\boldsymbol{r}_{ij}^{(k)} = <\{\mu_{ij1}^{(k)},\mu_{ij2}^{(k)},\cdots,\mu_{ij\#M_r}^{(k)}\},\{v_{ij1}^{(k)},v_{ij2}^{(k)},\cdots,v_{ij\#N_r}^{(k)}\}>$$

运用算法 1 将 t 个决策者给出的决策矩阵进行规范化处理，规范化后的决策矩阵为

$$\tilde{\boldsymbol{R}}^{(k)} = (\tilde{\boldsymbol{r}}_{ij}^{(k)})_{m\times n}, (k=1,2,\cdots,t)$$

步骤 2　计算 Pythagorean 犹豫模糊熵矩阵 $\boldsymbol{E}(\tilde{\boldsymbol{r}}_{ij}^{(k)})$。利用式(3)计算规范化后的决策矩阵 $\tilde{\boldsymbol{R}}^{(k)} = (\tilde{\boldsymbol{r}}_{ij}^{(k)})_{m\times n}$ 的 Pythagorean 犹豫模糊熵矩阵 $\boldsymbol{E}(\tilde{\boldsymbol{r}}_{ij}^{(k)})_{m\times n}$。

步骤 3　属性权重的确定。当属性权重完全未知时，运用式(5)计算属性权重；当属性权重部分已知时，运用式(6)中的模型计算属性权重。

步骤 4　对距离进行加权集结。利用加权平均算子和式(7)—式(8)计算各方案与正负理想点的加权平均距离。

步骤 5　计算贴近度。利用式(9)计算各方案的贴近度，并对其降序排列。

步骤 6　排序择优。对方案进行排序，并选出最优方案。

5　案例分析

减贫和脱贫是世界各国共同面临的难题，也是全球共同关注和研究的重大课题。针对我国的具体国情，习总书记提出了“脱贫贵在精准、重在精准、成败之举在于精准”的精准扶贫战略。十八届四中全会提出引入第三方评估机制，这是深化我国政府绩效改革。提高政府效能和完善政府治理体系的重要举措。第三方评估机制主要由三股力量来承担，一是高校的专家学者，二是商业运作的专业的管理咨询机构；三是乡镇级村干部代表[22]。

现考虑一个关于精准扶贫的多属性群决策问题。某村有 4 户人家申请扶贫基金补贴，但目前只有一个名额，即选择上述 4 户村民 x_1,x_2,x_3,x_4 中的一个，现由第三方评估的 3 位代表作为决策者 d_1,d_2,d_3 从下列 3 个准则出发对这 4 户村民进行评价：c_1：每年家庭的人均收入；c_2：家中的劳动力数量；c_3：家庭的恩格尔系数。设决策者的权重向量为：$L=\{0.3,0.5,0.2\}$，第 k 个决策者给出的 Pythagorean 犹豫模糊矩阵为 $\boldsymbol{R}^{(k)}(k=1,2,3)$ 如下：

$$\boldsymbol{R}^{(1)}=\begin{bmatrix} <\{0.3,0.4,0.5\},\{0.2,0.3\}> & <\{0.6,0.8\},\{0.1,0.2\}> & <\{0.4,0.6\},\{0.5,0.7\}> \\ <\{0.5,0.7\},\{0.1,0.2,0.3\}> & <\{0.6,0.8\},\{0.1,0.2\}> & <\{0.1,0.2\},\{0.4,0.6\}> \\ <\{0.2,0.3,0.6\},\{0.3,0.4\}> & <\{0.4,0.5\},\{0.3,0.4\}> & <\{0.5,0.7\},\{0.2,0.3\}> \\ <\{0.2,0.3\},\{0.2,0.4\}> & <\{0.4,0.6\},\{0.1,0.3\}> & <\{0.3,0.4\},\{0.3,0.4\}> \end{bmatrix}$$

$$\boldsymbol{R}^{(2)}=\begin{bmatrix} <\{0.4,0.5,0.6\},\{0.1,0.2\}> & <\{0.5,0.6\},\{0.1,0.3\}> & <\{0.7,0.8\},\{0.1,0.2\}> \\ <\{0.3,0.4\},\{0.2,0.4\}> & <\{0.6,0.7\},\{0.1,0.2\}> & <\{0.7,0.9\},\{0.1,0.2\}> \\ <\{0.3,0.5\},\{0.1,0.2,0.3\}> & <\{0.5,0.8\},\{0.3,0.4\}> & <\{0.6,0.7\},\{0.2,0.3\}> \\ <\{0.4,0.5\},\{0.3,0.4\}> & <\{0.6,0.8\},\{0.3,0.5\}> & <\{0.6,0.8\},\{0.3,0.4\}> \end{bmatrix}$$

$$\boldsymbol{R}^{(3)}=\begin{bmatrix} <\{0.3,0.4,0.5\},\{0.3,0.4\}> & <\{0.5,0.6\},\{0.1,0.3\}> & <\{0.5,0.6\},\{0.2,0.3\}> \\ <\{0.4,0.6\},\{0.2,0.3\}> & <\{0.6,0.7\},\{0.2,0.3\}> & <\{0.3,0.5\},\{0.6,0.7\}> \\ <\{0.3,0.4\},\{0.2,0.3,0.4\}> & <\{0.3,0.4\},\{0.1,0.3\}> & <\{0.4,0.6\},\{0.2,0.3\}> \\ <\{0.3,0.5\},\{0.3,0.5\}> & <\{0.4,0.6\},\{0.1,0.2\}> & <\{0.3,0.4\},\{0.2,0.4\}> \end{bmatrix}$$

步骤 1 利用算法 1,对矩阵 $\boldsymbol{R}^{(k)}(k=1,2,3)$ 做规范化处理:

$$\widetilde{\boldsymbol{R}}^{(1)}=\begin{bmatrix} <\{0.3,0.3,0.4,0.4,0.5,0.5\},\{0.2,0.2,0.2,0.3,0.3,0.3\}> & <\{0.6,0.8\},\{0.1,0.2\}> & <\{0.4,0.6\},\{0.5,0.7\}> \\ <\{0.5,0.5,0.5,0.7,0.7,0.7\},\{0.1,0.1,0.2,0.2,0.3,0.3\}> & <\{0.6,0.8\},\{0.1,0.2\}> & <\{0.1,0.2\},\{0.4,0.6\}> \\ <\{0.2,0.2,0.3,0.3,0.6,0.6\},\{0.3,0.3,0.3,0.4,0.4,0.4\}> & <\{0.4,0.5\},\{0.3,0.4\}> & <\{0.5,0.7\},\{0.2,0.3\}> \\ <\{0.2,0.2,0.2,0.3,0.3,0.3\},\{0.2,0.2,0.2,0.4,0.4,0.4\}> & <\{0.4,0.6\},\{0.1,0.3\}> & <\{0.3,0.4\},\{0.3,0.4\}> \end{bmatrix}$$

$$\widetilde{\boldsymbol{R}}^{(2)}=\begin{bmatrix} <\{0.4,0.4,0.5,0.5,0.6,0.6\},\{0.1,0.1,0.1,0.2,0.2,0.2\}> & <\{0.5,0.6\},\{0.1,0.3\}> & <\{0.7,0.8\},\{0.1,0.2\}> \\ <\{0.3,0.3,0.3,0.4,0.4,0.4\},\{0.2,0.2,0.2,0.4,0.4,0.4\}> & <\{0.6,0.7\},\{0.1,0.2\}> & <\{0.7,0.9\},\{0.1,0.2\}> \\ <\{0.3,0.3,0.3,0.5,0.5,0.5\},\{0.1,0.1,0.2,0.2,0.3,0.3\}> & <\{0.5,0.8\},\{0.3,0.4\}> & <\{0.6,0.7\},\{0.2,0.3\}> \\ <\{0.4,0.4,0.4,0.5,0.5,0.5\},\{0.3,0.3,0.3,0.4,0.4,0.4\}> & <\{0.6,0.8\},\{0.3,0.5\}> & <\{0.6,0.8\},\{0.3,0.4\}> \end{bmatrix}$$

$$\widetilde{\boldsymbol{R}}^{(3)}=\begin{bmatrix} <\{0.3,0.3,0.4,0.4,0.5,0.5\},\{0.3,0.3,0.3,0.4,0.4,0.4\}> & <\{0.5,0.6\},\{0.1,0.3\}> & <\{0.5,0.6\},\{0.2,0.3\}> \\ <\{0.4,0.4,0.4,0.6,0.6,0.6\},\{0.2,0.2,0.2,0.3,0.3,0.3\}> & <\{0.6,0.7\},\{0.2,0.3\}> & <\{0.3,0.5\},\{0.6,0.7\}> \\ <\{0.3,0.3,0.3,0.4,0.4,0.4\},\{0.2,0.2,0.3,0.3,0.4,0.4\}> & <\{0.3,0.4\},\{0.1,0.3\}> & <\{0.4,0.6\},\{0.2,0.3\}> \\ <\{0.3,0.3,0.3,0.5,0.5,0.5\},\{0.3,0.3,0.3,0.5,0.5,0.5\}> & <\{0.4,0.6\},\{0.1,0.2\}> & <\{0.3,0.4\},\{0.2,0.4\}> \end{bmatrix}$$

步骤 2 运用式(3)计算 $\widetilde{\boldsymbol{R}}^{(k)}$,$(k=1,2,3)$ 的熵矩阵 $\boldsymbol{E}(\tilde{r}_{ij}^{(k)})_{m\times n}$:

$$\boldsymbol{E}(\tilde{\boldsymbol{r}}_{ij}^{(k)})_{m\times n}=\begin{bmatrix} 0.8677 & 0.7138 & 0.8210 \\ 0.8785 & 0.6395 & 0.4979 \\ 0.9281 & 0.7566 & 0.7999 \end{bmatrix}$$

步骤 3 利用式(5)计算属性权重得到:

$$w_1=0.1482,w_2=0.4065,w_3=0.4452$$

步骤 4 计算方案 x_i 与正负理想点的距离:

正负理想点分别为

$$x^+=(r_{01}^{(k)+},r_{02}^{(k)+},r_{03}^{(k)+})=$$
$$(<\{1,1,1,1,1,1\},\{0,0,0,0,0,0\}>,<\{1,1,1,1\},\{0,0,0,0\}>,<\{1,1,1,1\},\{0,0,0,0\}>)$$

$$x^-=(r_{01}^{(k)-},r_{02}^{(k)-},r_{03}^{(k)-})=$$
$$(<\{0,0,0,0,0,0\},\{1,1,1,1,1,1\}>,<\{0,0,0,0\},\{1,1,1,1\}>,<\{0,0,0,0\},\{1,1,1,1\}>)$$

则方案 x_i 与正负理想点的加权距离为

$$AD_1^+=0.6060,AD_2^+=0.5913,$$
$$AD_3^+=0.6510,AD_4^+=0.6433,$$
$$AD_1^-=0.8080,AD_2^-=0.8036,$$
$$AD_3^-=0.8179,AD_4^-=0.8002。$$

步骤 5 计算方案 x_i 贴近度 P_i 得到:

$$P_1=0.5714,P_2=0.5761,$$
$$P_3=0.5568,P_4=0.5543。$$

步骤 6 排序和择优。

贴近度排序为:$P_2>P_1>P_3>P_4$,故方案的排序 $x_2>x_1>x_3>x_4$,即第二户农户符合扶贫基金补贴政策。

6 结　论

针对模糊信息下的决策问题,提出了一种基于 Pythagorean 犹豫模糊熵的多属性群决策方法。首先,在之前的研究基础上,构造基于 Pythagorean 犹豫模糊环境下熵的公理化定义及计算公式,其次,为解决任意多个 Pythagorean 犹豫模糊数之间隶属度集合和非隶属度集合长度不相等的问题,利用最小公倍数扩充法对 Pythagorean 犹豫模糊集进行规范化处理,并在之前的基础上,完善了最小公倍数扩充规则的定义及算法;之后,以 Pythagorean 犹豫模糊熵作为决策信息差异程度的度量,给出属性权重完全未知或部分已知情况下权重的确定方法;最后,以扶贫项目为背景,实例论证了提出的 Pythagorean 犹豫模糊熵的多属性群决策方法是可行且有效的。未来,Pythagorean 犹豫模糊熵可被运用到模糊模式识别、医疗诊断、不确定性决策问题中。

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An Approach to Multiple Attribute Group Decision Making Based on the Pythagorean Hesitant Fuzzy Entropy

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Abstract: For the decision-making problems with vague information, this paper presents a multiple attribute group decision making approach based on Pythagorean hesitant fuzzy entropy. Firstly, the axiomatic definition and calculation formula of Pythagorean hesitant fuzzy entropy are proposed. Classical normalization method may cause the loss of original information, in order to overcome the shortcoming, a least common multiple extended method is completed to normalize the Pythagorean hesitant fuzzy sets. This expansion method can effectively keep the original information. Then, taking Pythagorean hesitant fuzzy entropy as the difference degree of decision information, a multi-attribute group decision making method is given to determine the weight when the attribute weight is completely unknown or partially known. Meanwhile, the distance and entropy measures of Pythagorean hesitant fuzzy numbers are put forward. Finally, an approach of TOPSIS based on Pythagorean hesitant fuzzy entropy is developed. At the same time, a numerical example of precision poverty alleviation project is provided to illustrate the feasibility and effectiveness of the proposed method.

Key words: multi-attribute group decision making; Pythagorean hesitant fuzzy entropy; attribute weight; least common multiple expansion principle; TOPSIS

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