doi:10.16055/j.issn.1672-058X.2015.0005.009

二重非齐次马氏链的一个重要结论*

桂春燕, 胡永模
(安庆师范学院 数学与计算科学学院,安徽 安庆 246133)

摘　要:马尔科夫链是一类特殊的随机过程,相对于一重马氏链,二重马氏链的应用也十分广泛,主要给出了二重非齐次马氏链的一个重要结论.

关键词:二重非齐次马氏链;收敛;函数

中图分类号:O211.4　　**文献标识码**:A　　**文章编号**:1672-058X(2015)05-0031-03

设$\{f_n(x,y,z),n\in \mathbf{N}_+\}$是定义在 $E\times E\times E$ 上的一列三元实值函数,记 $\xi_n=f_n(X_n,X_{n+1},X_{n+2})$,则 ξ_n 为二重马氏链$\{X_n,n\in \mathbf{N}_+\}$的一列函数,$\{a_n\}$为一数列,且 $0<a_n\uparrow$,$n\in \mathbf{N}_+$.定义

$$f_n^*(x,y,z)=\begin{cases} f_n(x,y,z), & \text{若 } |f_n(x,y,z)|\leqslant a_n \\ 0, & \text{反之} \end{cases}$$

设 $r,s\in E$,λ 为实数,若 $P(X_{n+1}=s,X_n=r)>0$,则令

$$\eta_n(r,s)=E[f_n^*(X_n,X_{n+1},X_{n+2})\mid X_n=r,X_{n+1}=s] \tag{1}$$

$$\zeta_n(\lambda,r,s)=E\left\{\exp\left[\frac{\lambda f_n^*(X_n,X_{n+1},X_{n+2})-\eta_n(r,s)}{a_n}\right]\mid X_n=r,X_{n+1}=s\right\}=$$
$$\sum_{k=0}^{+\infty}P_n(rs,k)\exp\left[\frac{\lambda f_n^*(r,s,k)-\eta_n(r,s)}{a_n}\right] \tag{2}$$

显然有

$$\left|\frac{f_n^*(X_n,X_{n+1},X_{n+2})-\eta_n(r,s)}{a_n}\right|\leqslant 2$$

定理 1　令$\{\xi_n=f_n(X_n,X_{n+1},X_{n+2}),n\in \mathbf{N}_+\}$为二重非齐次马氏链$\{X_n,n\in \mathbf{N}_+\}$的函数列,$\{\varphi_n,n\in \mathbf{N}_+\}$是 $\mathbf{R}$ 上正值连续偶函数,且当$|x|\uparrow$时,$\dfrac{\varphi_n(x)}{|x|}\uparrow$,$\dfrac{\varphi_n(x)}{x^2}\downarrow$,令$\{a_n,n\in \mathbf{N}_+\}$为一数列,且 $0<a_n\uparrow$,又

$$\sum_{n=0}^{\infty}E\frac{\varphi_n(\xi_n)}{\varphi_n(a_n)}<\infty \tag{3}$$

则有 $\displaystyle\sum_{m=0}^{n-2}\frac{\xi_m-E(\xi_m\mid X_{m-k+1},X_{m-k+2})}{a_m}$　a.e.收敛.

证明　当 $k=1$ 时,根据式(3)可得

$$\sum_{n=0}^{\infty}E\left\{\frac{E[\varphi_n(\xi_n)\mid X_n,X_{n+1}]}{\varphi_n(a_n)}\right\}<\infty \tag{4}$$

收稿日期:2014-07-15;**修回日期**:2014-10-08.

* **基金项目**:安庆师范学院校级青年科研项目(KJ201106).

作者简介:桂春燕(1982),女,讲师,硕士,从事马氏链研究.

而 $E[\varphi_n(\xi_n)|X_n,X_{n+1}](n\in\mathbf{N}_+)$ 是非负随机变量,则有 $\sum_{m=0}^{n-2}\frac{E[\varphi_m(\xi_m)|X_m,X_{m+1}]}{\varphi_m(a_m)}$ a.e.收敛,

$$\sum_{m=0}^{\infty}P(\xi_m\neq f_m^*(X_m,X_{m+1},X_{m+2}))=\sum_{m=0}^{\infty}\int_{|\xi_m|>a_m}P(d\omega)\leqslant\sum_{m=0}^{\infty}\int_{|\xi_m|>a_m}\frac{\varphi_m(\xi_m)}{\varphi_m(a_m)}P(d\omega)\leqslant$$
$$\sum_{m=0}^{\infty}\int_{|\xi_m|>a_m}\frac{\varphi_m(\xi_m)}{\varphi_m(a_m)}P(d\omega)\leqslant\tag{5}$$
$$\frac{E[\varphi_m(\xi_m)]}{\varphi_m(a_m)}<\infty$$

再由 B-C 引理得 $\xi_m\neq f_m^*(X_m,X_{m+1},X_{m+2})$ 仅对有限个状态成立,则 $\sum_{m=0}^{n-2}\frac{\xi_m-f_m^*(X_m,X_{m+1},X_{m+2})}{a_m}$ a.e.收敛.

由式(1)和(2),得

$$E\left\{\frac{\lambda f_n^*(X_n,X_{n+1},X_{n+2})-\eta_n(r,s)}{a_n}|X_n=r,X_{n+1}=s\right\}=0\tag{6}$$

$$\zeta_n(\lambda,r,s)-1=$$
$$E\left\{\exp\left[\frac{\lambda f_n^*(X_n,X_{n+1},X_{n+2})-\eta_n(r,s)}{a_n}\right]|X_n=r,X_{n+1}=s\right\}-1=$$
$$E\left\{\exp\left[\frac{\lambda f_n^*(X_n,X_{n+1},X_{n+2})-\eta_n(r,s)}{a_n}\right]-1-\frac{\lambda f_n^*(X_n,X_{n+1},X_{n+2})-\eta_n(r,s)}{a_n}|X_n=r,X_{n+1}=s\right\}\tag{7}$$

$$|\eta_n(r,s)|=$$
$$|E[f_n^*(X_n,X_{n+1},X_{n+2})|X_n=r,X_{n+1}=s]|=\left|\int_{-\infty}^{\infty}f_n^*(r,s,X_{n+2})\mathrm{d}F_{X_{n+2}|X_{n+1},X_n}(x|r,s)\right|\leqslant$$
$$\int_{-\infty}^{\infty}|f_n^*(r,s,X_{n+2})|\mathrm{d}F_{X_{n+2}|X_{n+1},X_n}(x|r,s)=\int_{-\infty}^{\infty}|f_n^*(X_n,X_{n+1},X_{n+2})|\mathrm{d}F_{X_{n+2}|X_{n+1},X_n}(x|r,s)\tag{8}$$

其中 $F_{X_{n+2}|X_{n+1},X_n}(x|r,s)=P(X_{n+2}\leqslant x|X_n=r,X_{n+1}=s)$.而当 $|x|<t$ 时,$0\leqslant e^x-1-x\leqslant x^2e^t$,故

$$0\leqslant E\left\{\exp\left[\frac{\lambda f_n^*(X_n,X_{n+1},X_{n+2})-\eta_n(r,s)}{a_n}\right]-1-\frac{\lambda f_n^*(X_n,X_{n+1},X_{n+2})-\eta_n(r,s)}{a_n}|X_n=r,X_{n+1}=s\right\}=$$
$$\zeta_n(\lambda,r,s)-1\leqslant e^{2|\lambda|}E\left\{\left[\frac{\lambda f_n^*(X_n,X_{n+1},X_{n+2})-\eta_n(r,s)}{a_n}\right]^2|X_n=r,X_{n+1}=s\right\}=$$
$$e^{2|\lambda|}\lambda^2E\left\{\frac{(f_n^*(X_n,X_{n+1},X_{n+2}))^2+(\eta_n(r,s))^2-2f_n^*(X_n,X_{n+1},X_{n+2})\eta_n(r,s)}{a_n^2}|X_n=r,X_{n+1}=s\right\}\leqslant$$
$$e^{2|\lambda|}\lambda^2E\left\{\frac{(f_n^*(X_n,X_{n+1},X_{n+2}))^2+(\eta_n(r,s))^2}{a_n^2}|X_n=r,X_{n+1}=s\right\}=$$
$$2e^{2|\lambda|}\lambda^2E\left\{\frac{(f_n^*(X_n,X_{n+1},X_{n+2}))^2}{a_n^2}|X_n=r,X_{n+1}=s\right\}\leqslant$$
$$2e^{2|\lambda|}\lambda^2E\left\{\frac{\varphi_n(f_n^*(X_n,X_{n+1},X_{n+2}))}{\varphi_n(a_n)}|X_n=r,X_{n+1}=s\right\}\tag{9}$$

由式(4)和式(9)知 $\sum_{m=0}^{n-2}(\zeta_n(\lambda,X_m,X_{m+1})-1)$ a.e.收敛,从而 $\sum_{m=0}^{n-2}\zeta_n(\lambda,X_m,X_{m+1})$ a.e.收敛.于是

$$\lim_{n\to\infty}\exp\left\{\lambda\sum_{m=0}^{n-2}\left[\frac{f_m^*(X_m,X_{m+1},X_{m+2})-\eta_m(r,s)}{a_m}\right]\right\}\tag{10}$$

存在且 a.e.有限.分别令 $\lambda=-1,\lambda=1$,代入式(9)可推得 $\sum_{m=0}^{n-2}\frac{1}{a_m}[f_m^*(X_m,X_{m+1},X_{m+2})-\eta_m(r,s)]$ a.e. 有限,由已知若 $|x|>a_n$,则 $\frac{|x|}{a_n}<\frac{\varphi_n(x)}{\varphi_n(a_n)}$,从而

$$\begin{aligned}&\left|\frac{E(\xi_n|X_n,X_{n+1})-E(f_n^*(X_n,X_{n+1},X_{n+2})|X_n,X_{n+1})}{a_n}\right|=\\&\left|\frac{E(\xi_n-f_n^*(X_n,X_{n+1},X_{n+2})|X_n,X_{n+1})}{a_n}\right|\leqslant\\&E\left(\frac{|\xi_n-f_n^*(X_n,X_{n+1},X_{n+2})|}{a_n}|X_n,X_{n+1}\right)=\\&E\left(\frac{|\xi_n|}{a_n}\chi(|\xi_n|>a_n)|X_n,X_{n+1}\right)\leqslant\\&E\left(\frac{\varphi_n(\xi_n)}{\varphi_n(a_n)}\chi(|\xi_n|>a_n)|X_n,X_{n+1}\right)\leqslant\\&\frac{E(\varphi_n(\xi_n)|X_n,X_{n+1})}{\varphi_n(a_n)}\end{aligned}\tag{11}$$

于是 $\sum_{m=0}^{n-2}\frac{E(\xi_m|X_m,X_{m+1})-E(f_m^*(X_m,X_{m+1},X_{m+2})|X_m,X_{m+1})}{a_m}$ a.e. 收敛,知 $\sum_{m=0}^{n-2}\frac{\xi_m-E(\xi_m|X_m,X_{m+1})}{a_m}$ a.e. 收敛. 故 $k=1$ 时,结论成立;当 $k>1$ 时,$\{X_{nk+m},n\geqslant0\}$ 也是二重非齐次马氏链,同理可以证明结论成立.

参考文献:

[1] 刘文,杨卫国.可列非齐次马氏链的若干极限定理[J].应用数学学报,1992,15(4):479-489

[2] 刘文,杨卫国.非齐次马氏链二元泛函的若干极限性质[J].河北工业大学学报,1995,18(3):439-445

[3] BILLINGSLEY P.Probability and Measure[M].New York:Wiley,1986

An Important Conclusion of Nonhomogeneous Tow-order Markov Chains

GUI Chun-Yan, HU Yong-mo

(School of Mathematics and Computational Science, Anqing Normal University, Anqing 246133, China)

Abstract: Markov chain is a special kind of stochastic process. Relative to one-order Markov chains, the application of the tow-order Markov chains is also widely applied. An important conclusion of nonhomogeneous tow-order Markov chains is given.

Key words: nonhomogeneous tow-order Markov chains; convergence; function