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Stability Properties and Existence of Asymptotically Almost Periodic Solutions of Volterra Difference Equations*

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Abstract: For nonlinear Volterra difference equations with unbounded delay, we characterize their existence of totally stable and asymptotically almost periodic solution by using certain total stability properties of a bounded solution.

Key words: Volterra difference equations; asymptotically almost periodic solutions; total stability properties

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1 Introduction

Recently, the existence of almost periodic solutions of difference systems with delay has been treated in several works[1-7]. Y. Song and H. Tian[8] have studied the existence of periodic and almost periodic solutions for nonlinear Volterra difference equations by means of certain stability of a bounded solution. Hamaya[9] has investigated the existence of almost periodic solutions of functional difference equations with infinite delay by using stability properties of bounded solutions. Y. Xia and S. Cheng[10] have studied the existence of almost periodic solutions of ordinary difference equation by using globally quasi-uniformly asymptotically stable. In [11], Hamaya discussed the relationships between the stability properties of solutions of the nonlinear integrodifferential equation and those in their limiting equations. In [12], Hamaya discussed the existence of a periodic solution of an integrodifferential equation by using stability properties of a bounded solution. In this paper, we shall concern with nonlinear Volterra difference equations with unbounded delay and discuss the existence of totally stable and asymptotically almost periodic solution by using certain total stability properties of a bounded solution.

Let R^m denote Euclidean m-space, Z is the set of integers, Z^+ is the set of nonnegative integers and $|\cdot|$ will denote the Euclidean norm in R^m, for any interval $I \subset Z := (-\infty, +\infty)$, we denote by $BS(I)$ the set of all bounded functions mapping I into R^m, and $|\phi|_I = \sup\{|\phi(s)| : s \in I\}$.

Now, for any function $x:(-\infty, a) \to R^m$ and $n < a$, define a function $x_n : Z^- = (-\infty, 0] \to R^m$ by $x_n(s) = x(n+s)$ for $s \in Z^-$. Let BS be a real linear space of functions mapping Z^- into R^m with sup-norm:

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$$BS = \{\phi \mid \phi: Z^- \to R^m \quad \text{with} \quad |\phi|_\infty = \sup_{s \in Z^-} |\phi(s)| < \infty\}$$

We introduce an almost periodic function $f(n,x): Z \times D \to R^m$, where D is an open set in R^m.

Definition 1　$f(n,x)$ is said to be almost periodic in n uniformly for $x \in D$, if for any $\varepsilon > 0$ and any compact set K in D, there exists a positive integer $L^*(\varepsilon,k)$ such that any interval of length $L^*(\varepsilon,k)$ contains an integer τ for which

$$|f(n+\tau,x) - f(n,x)| \leqslant \varepsilon$$

for all $n \in Z$ and all $x \in K$, such a number τ in above inequality is called an ε-translation number of $f(n,x)$.

In order to formulate a property of almost periodic functions, which is equivalent to the above definition, we discuss the concept of the normality of almost periodic functions. Namely, Let $f(n,x)$ be almost periodic in n uniformly for $x \in D$, then, for any sequence $\{h'_k\} \subset Z$, there exists a subsequence $\{h_k\}$ of $\{h'_k\}$ and function $g(n,x)$ such that

$$f(n+h_k,x) \to g(n,x) \tag{1}$$

uniformly on $Z \times K$ as $K \to \infty$, where K is a compact set in D. There are many properties of the discrete almost periodic functions[10,13], which are corresponding properties of the continuous almost periodic functions $f(n,x) \in C(R \times D, R^m)$. We shall denote by $T(f)$ the function space consisting of all translates of f, that is, $f_\tau \in T(f)$, where

$$f_\tau(n,x) = f(n+\tau,x), \quad \tau \in Z \tag{2}$$

Let $H(f)$ denote the uniform closure of $T(f)$ in the sense of (2), $H(f)$ is called the hull of f. In particular, we denote by $\Omega(f)$ the set of all limit functions $g \in H(f)$ such that for some sequence $\{n_k\}$, $n_k \to \infty$ as $K \to \infty$ and $f(n+n_k,x) \to g(n,x)$ uniformly on $Z \times S$ for any compact subset S in R^m. By (1), if $f: Z \times D \to R^m$ is almost periodic in n uniformly for $x \in D$, so is a function in $\Omega(f)$. The following concept of asymptotic almost periodicity was introduced by Frechet in the case of continuous function.

Definition 2　$u(n)$ is said to be asymptotically almost periodic if it is a sum of a almost periodic function $p(n)$ and a function $q(n)$ defined on $I^* = [0,\infty)$ which tends to zero as $n \to \infty$, that is,

$$u(n) = p(n) + q(n)$$

$u(n)$ is asymptotically almost periodic if and only if for any sequence $\{n_k\}$ such that $n_k \to \infty$ as $k \to \infty$, there exists a subsequence $\{n_k\}$ for which $u(n+n_k)$ converges uniformly on $0 \leqslant n < \infty$.

2　Preliminaries

We consider a system of Volterra difference equation

$$x(n+1) = f(n+n_k, x(n)) + \sum_{s=-\infty}^{0} F(n+n_k, s, x(n+s), x(n)) \tag{3}$$

where $f: Z \times R^m \to R^m$ is continuous at second variable $x \in R^m$ and $F(n,s,x,y)$ is defined for $n \in Z$, $s \in (-\infty, 0]$, $x \in R^m$ and $y \in R^m$, and continuous for $x \in R^m$ and $y \in R^m$.

We impose the following assumptions on Eq. (3):

(H1) $f(n,x)$ is an almost periodic in n uniformly for $x \in R^m$, and $F(n,s,x,y)$ is almost periodic in n uniformly for $(s,x,y) \in k^*$, that is for any $\varepsilon > 0$ and any compact set k^*, there exists an integer $L^* = L^*(c,k) > 0$ such that any interval of length L^* contains a τ for which

$$|F(n+\tau,s,x,y)-F(n,s,x,y)|\leqslant\varepsilon$$

for all $n\in Z$ and all $(s,x,y)\in k^*$.

(H2) For any $\varepsilon>0$ and any $r>0$, there exists an $S=S(\varepsilon,r)>0$ such that

$$\sum_{s=-\infty}^{-s}|F(n,s,x(n+s),x(n))|\leqslant\varepsilon$$

for all $n\in Z$, whenever $|x(\delta)|\leqslant r$ for all $\delta\leqslant n$.

(H3) Eq. (3) has a bounded solution $u(n)$ defined on $[0,\infty)$ which passes through $(0,u_0)$, that is $\sup_{n\geqslant 0}|u(n)|<\infty$ and $u_0\in BS$.

For any $\theta,\psi\in BS$, we set $\rho(\theta,\psi)=\sum_{j=1}^{\infty}\rho_j(\theta,\psi)/[2^j(1+\rho_j(\theta,\psi))]$, where $\rho_j(\theta,\psi)=\sup_{-j\leqslant s\leqslant 0}|\theta(s)-\psi(s)|$, clearly, $\rho(\theta_n,\theta)\to 0$ as $n\to\infty$ if and only if $\theta_n(s)\to\theta(s)$ uniformly on any compact subset of $(-\infty,0]$ as $n\to\infty$. We denote by (BS,ρ) the space of bounded functions $\phi:(-\infty,0]\to R^m$ with metric ρ.

Let K be the compact set in R^m such that $u(n)\in k$ for all $n\in Z$, where $u(n)=\phi^0(n)$ for $n\leqslant 0$.

Definition 3 A bounded solution $u(n)$ of Eq. (3) is said to be

(i) (k_0,ρ)-stable (in short, $(k_0,\rho)-$S) if for any $\varepsilon>0$ there exists a $\delta(n_0,\varepsilon)>0$ such that if $n_0\geqslant 0$, $\rho(x_{n_0},u_{n_0})<\delta(n_0,\varepsilon)$, then $\rho(x_n,u_n)<\varepsilon$ for all $n\geqslant n_0$, where $x(n)$ is a solution of (3) through (n_0,ϕ) such that $x_{n_0}(s)=\phi(s)\in k$ for all $s\leqslant 0$.

(ii) (k_0,ρ)-uniformly stable (in short, $(k_0,\rho)-$US) if for any $\varepsilon>0$ there exists a $\delta(\varepsilon)>0$ such that if $n_0\geqslant 0$, $\rho(x_{n_0},u_{n_0})<\delta(\varepsilon)$, then $\rho(x_n,u_n)<\varepsilon$ for all $n\geqslant n_0$, where $x(n)$ is a solution of (3) through (n_0,ϕ) such that $x_{n_0}(s)=\phi(s)\in k$ for all $s\leqslant 0$.

(iii) (k_0,ρ)-equi asymptotically stable (in short, $(k_0,\rho)-$EAS) if it is $(k_0,\rho)-$S and for any $\varepsilon>0$, there exists a $\delta_0(n_0)>0$ and a $T(n_0,\varepsilon)>0$ such that if $n_0\geqslant 0$, $\rho(x_{n_0},u_{n_0})<\delta_0(n_0)$, then $\rho(x_n,u_n)<\varepsilon$ for all $n\geqslant n_0+T(\varepsilon)$, where $x(n)$ is a solution of (3) through (n_0,ϕ) such that $x_{n_0}(s)=\phi(s)\in k$ for all $s\leqslant 0$.

(iv) (k_0,ρ)-uniformly asymptotically stable (in short, $(k_0,\rho)-$UAS) if it is $(k_0,\rho)-$US and is (k_0,ρ)-quasi uniformly asymptotically stable, that is, if the δ_0 and the T in above (iii) are independent of n_0 (for any $\varepsilon>0$ there exists a $\delta_0>0$ and a $T(\varepsilon)>0$ such that if $n_0\geqslant 0$, $\rho(x_{n_0},u_{n_0})<\delta_0$, then $\rho(u_n,u_n)<\varepsilon$ for all $n\geqslant n_0+T(\varepsilon)$, where $x(n)$ is a solution of (3) through (n_0,ϕ) such that $x_{n_0}(s)=\phi(s)\in k$ for all $s\leqslant 0$.).

(v) (k_0,ρ)-eventually totally stable (in short, $(k_0,\rho)-$ETS), if for any $\varepsilon>0$, there exists a $\delta(\varepsilon)>0$ and $\alpha(\varepsilon)$ such that if $n_0\geqslant\alpha(\varepsilon)$, $\rho(x_{n_0},u_{n_0})<\delta(\varepsilon)$ and $h\in BS([n_0,\infty))$ which satisfies $|h|_{[n_0,\infty)}<\delta(\varepsilon)$, then $\rho(x_{n_0},u_{n_0})<\delta(\varepsilon)$ for all $n\geqslant n_0$, where $x(n)$ is a solution of

$$x(n+1)=f(n+n_k,x(n))+\sum_{s=-\infty}^{0}F(n+n_k,s,x(n+s),x(n))+h(n)$$

through (n_0,ϕ) such that $x_{n_0}(s)=\phi(s)\in k$ for all $s\leqslant 0$. If we can choose $\alpha(\varepsilon)\equiv 0$, then $u(n)$ is said to be (k_0,ρ)-totally stable (in short, $(k_0,\rho)-$TS). In the case where $h(n)\equiv 0$, this gives the definition of the $(k_0,\rho)-$US of $u(n)$.

When we restrict solutions x to those which remain in k_0, that is, $x(n)\in k_0$ for all $n\geqslant n_0$, we say that $u(n)$ is relatively eventually totally (k_0,ρ)-stable (in short, $(k_0,\rho)-$RETS) and so on.

3 Main Results

Theorem 1 Under the assumptions (A) through (D), if the bounded solution $u(n)$ of (3) is (k_0,ρ)-RETS, then $u(n)$ is asymptotically almost periodic.

Proof Let $\{n_k\}$ be a sequence such that $n_k\to\infty$ as $k\to\infty$. If we set $u^k(n)=u(n+n_k)$, $k=1,2,\cdots$, $u^k(n)$ is a solution of

$$x(n+1)=f(n+n_k,x(n))+\sum_{s=-\infty}^{0}F(n+n_k,s,x(n+s),x(n))$$

and $u^k(n)$ remains in k_0. Since $u(n)$ is relatively eventually totally (k_0,ρ)-stable, $u^k(n)$ is also relatively eventually totally (k_0,ρ)-stable with the same $(\delta(\cdot),\alpha(\cdot))$ as for $u(n)$.

For given $\varepsilon>0$, there exists a positive integer $k_1(\varepsilon)$ such that $n_k\geqslant\alpha(\varepsilon)$ if $k\geqslant k_1(\varepsilon)$. Taking a subsequence if necessary, we can assume that $u^k(n)$ converges uniformly on any compact set in $(-\infty,0]$ as $k\to\infty$. Therefore there exists a positive integer $k_2(\varepsilon)$ such that if $k,m\geqslant k_2(\varepsilon)$, $\rho(u_0^k,u_0^m)<\delta(\varepsilon)$. Clearly $u^m(n)=u(n+n_m)$ is a solution of

$$x(n+1)=f(n+n_k,x(n))+\sum_{s=-\infty}^{0}F(n+n_k,s,x(n+s),x(n))+h(n) \tag{4}$$

and $u^m(n)\in k_0$ for all $n\in Z$, where

$$h(n)=f(n+n_m,u^m(n))+\sum_{s=-\infty}^{0}F(n+n_m,s,u^m(n+s),u^m(n))-$$

$$f(n+n_k,u^m(n))+\sum_{s=-\infty}^{0}F(n+n_k,s,u^m(n+s),u^m(n))$$

We shall show that there exists a positive integer $k_0(\varepsilon)$ such that if $k,m\geqslant k_0(\varepsilon)$, $|h(n)|<\delta(\varepsilon)$ for $n\geqslant 0$, there exists a $c>0$ such that $|x|\leqslant c$ for all $x\in k_0$. It is clear that $u^k(n)\leqslant c$ and $u^m(n)\leqslant c$ for all $n\in Z$. By assumption (H2), there exists an $S=S(c,\varepsilon)>0$ such that

$$\sum_{s=-\infty}^{-s}F(n+n_m,s,u^m(n+s),u^m(n))\leqslant\delta(\varepsilon)/5 \qquad \text{for all } n\in Z$$

$$\sum_{s=-\infty}^{-s}F(n+n_k,s,u^m(n+s),u^m(n))\leqslant\delta(\varepsilon)/5 \qquad \text{for all } n\in Z$$

Since $f(n,x)$ and $F(n,s,x,y)$ are almost periodic in t for this S, there exists a positive integer $k_0(\varepsilon)\geqslant\max(k_1(\varepsilon),k_2(\varepsilon))$ such that if $k,m\geqslant k_0(\varepsilon)$.

$$|F(n+n_m,s,u^m(n+s),u^m(n))-F(n+n_k,s,u^m(n+s),u^m(n))|<\delta(\varepsilon)/5s$$
$$\text{for all } n\in Z \text{ and } -s\leqslant s<0$$

$$|f(n+n_m,u^m(n))-f(n+n_k,u^m(n))|<\delta(\varepsilon)/5 \text{ for all } n\in Z$$

Since we have

$$\left|\sum_{s=-\infty}^{0}F(n+n_m,s,u^m(n+s),u^m(n))-\sum_{s=-\infty}^{0}F(n+n_k,s,u^m(n+s),u^m(n))\right|\leqslant$$

$$\sum_{s=-\infty}^{-s}|F(n+n_m,s,u^m(n+s),u^m(n))|-\sum_{s=-\infty}^{-s}|F(n+n_k,s,u^m(n+s),u^m(n))|-$$

$$\sum_{s=-s}^{0} | F(n+n_m,s,u^m(n+s),u^m(n)-F(n+n_k,s,u^m(n+s),u^m(n))) |$$

We obtain $|h(n)|<\delta(\varepsilon)$ for $n\geqslant 0$ if $k,m\geqslant k_0(\varepsilon)$. Since $u^m(n)$ is a solution of (4) which remains in k_0 and $u^k(n)$ is relatively eventually totally (k_0,ρ)-stable, we have $\rho(u_n^k,u_n^m)<\varepsilon$ for all $n\geqslant 0$ if $k,m\geqslant k_0(\varepsilon)$. This implies that if $k,m\geqslant k_0(\varepsilon)$,

$$|u(n+n_k)-u(n+n_m)| \leqslant \sup_{s\in[-1,0]} |u(n+n_k+s)-u(n+n_m+s)| < 4\varepsilon$$

for all $\varepsilon\leqslant 1/4$ and all $n\geqslant 0$. Thus we see that for any sequence $\{n_k\}$ such that $n_k\to\infty$ as $k\to\infty$, there exists a subsequence $\{n_{k_j}\}$ of $\{n_k\}$ for which $u(n+n_{k_j})$ converges uniformly on $[0,\infty)$ as $j\to\infty$. This shows that $u(n)$ is asymptotically almost periodic in n.

We donte by $\Omega(f,F)$ the set of all limit functions $(g(n,x),G(n,s,x,y))$ such that for some sequence $\{n_k\}$, $n_k\to\infty$ as $k\to\infty$, $f(n+n_k,x)$ converges to $g(n,x)$ uniformly on $Z\times S$ for any compact subset S in R^n and $F(n+n_k,s,x,y)$ converges to $G(n,s,x,y)$ uniformly on $Z\times S^*$ for any compact subset S^* in Z^*. Moreover, we denote by $(v,g,G)\in\Omega(u,f,F)$ when for the same sequence $\{n_k\}$, $u(n+n_k)\to v(n)$ uniformly on any compact subset in Z as $k\to\infty$. Then a system

$$x(n+1)=g(n,x(n))+\sum_{s=-\infty}^{0} G(n,s,x(n+s),x(n)) \tag{5}$$

is called a limiting equation of (3) when $(g,G)\in\Omega(f,F)$ and $v(n)$ is a solution of (5) when $(v,g,G)\in\Omega(u,f,F)$.

In the followings, we let K be the compact set such that $k=\overline{N(\varepsilon_0,k_0)}$ for some $\varepsilon_0>0$, where $\overline{N(\varepsilon_0,k_0)}$ denotes the closure of the ε_0-neighborhood $N(\varepsilon_0,k_0)$ of k_0.

Theorem 2 Under the assumptions (A) through (D), assume that system (3) admits a limiting equation (5) whose solution $v(n)$ such that $(v,g,G)\in\Omega(u,f,F)$ is (k_0,ρ) –TS, then $u(n)$ is (k_0,ρ) –ETS.

Proof Since $(v,g,G)\in\Omega(u,f,F)$, there exists a sequence $\{n_k\}$, $n_k\to\infty$ as $k\to\infty$, such that $f(n+n_k,x)\to g(n,s)$ uniformly on $Z\times K$, $F(n+n_k,s,x,y)\to G(n,s,x,y)$ uniformly on $Z\times S^*\times K\times K$ for any compact set S^* in $(-\infty,0]$ and $u(n+n_k)\to v(n)$ uniformly on any compact set in Z as $k\to\infty$. If we set $u^k(n)=u(n+n_k)$, $k=1,2,\cdots$, $u^k(n)$ is a solution of

$$x(n+1)=f(n+n_k,x(n))+\sum_{s=-\infty}^{0} F(n+n_k,s,x(n+s),x(n)) \tag{6}$$

and $u_0^k(s)\in k_0$ for all $s\leqslant 0$. There exists a $c>0$ such that $|x|\leqslant c$ for all $x\in k$. Let $x(\delta)$ be a continuous function such that $x(\delta)\in k$ for all $\delta\leqslant n$. By assumption (H2), there exists an $S=S(c,\varepsilon)>0$ such that

$$\sum_{s=-\infty}^{-s} | F(n+n_k,s,x(n+s),x(n)) | \leqslant \delta(\delta(\varepsilon/2)/2)/5$$

where $\delta(\cdot)$ is the number for the total (k_0,ρ)-stability of $v(n)$. Since $F(n+n_k,s,x,y)\to G(n,s,x,y)$, we have

$$\sum_{s=-\infty}^{-s} | G(n,s,x(n+s),x(n)) | \leqslant \delta(\delta(\varepsilon/2)/2)/5$$

for the same S. Thus, by the same argument in the proof of Theorem 1, there exists a positive integer $K_0(\varepsilon)$ such that if $K\geqslant K_0(\varepsilon)$,

$$|f(n+n_k,x(n))-\sum_{s=-\infty}^{0}F(n+n_k,s,x(n+s),x(n))-$$

$$g(n,x(n))-\sum_{s=-\infty}^{0}G(n,s,x(n+s),x(n))|<\delta(\delta(\varepsilon/2)/2) \tag{7}$$

and $\rho(u_0^k,v_0)<\delta(\delta(\varepsilon/2)/2)$. Since $u^k(n)$ is a solution of (6), $u^k(n)$ is a solution of

$$x(n+1)=g(n,x(n))-\sum_{s=-\infty}^{0}G(n,s,x(n+s),x(n))$$

where $u_0^k(s)\in k_0$ for all $s\leqslant 0$. Since $\rho(u_0^k,v_0)<\delta(\delta(\varepsilon/2)/2)$, by (7) and if $K\geqslant K_0(\varepsilon)$ and since $v(n)$ is totally (k_0,ρ)-stable, we have

$$\rho(u_n^k,v_n)<\delta(\varepsilon/2)/2 \qquad \text{for all } t\geqslant 0 \tag{8}$$

We let $m=K_0(\varepsilon)$ and $\alpha(\varepsilon)=n_m$. We shall show that if $n_0\geqslant\alpha(\varepsilon)$, $\rho(u_{n_0},y_{n_0})<\delta(\varepsilon/2)/2$ and $|h(n)|<\delta(\varepsilon/2)/2$ for $n\geqslant n_0$, where $y(n)$ is a solution of

$$x(n+1)=f(n,x(n))-\sum_{s=-\infty}^{0}F(n,s,x(n+s),x(n))+h(n)$$

such that $y_{n_0}(s)\in k_0$ for all $s\leqslant 0$. Now we assume that $\rho(u_\delta,y_\delta)=\varepsilon$ for a $\delta>n_0$ and $\rho(u_n,y_n)<\varepsilon$ for $n_0\leqslant n<\delta$, if we set $z(n)=y(n+n_m)$, $z(n)$ is a solution defined on $n_0-n_m\leqslant n\leqslant\delta-n_m$ of

$$x(n+1)=f(n+n_m,x(n))+\sum_{s=-\infty}^{0}F(n+n_m,s,x(n+s),x(n))+h(n+n_m)$$

such that $z_{n_0-n_m}(s)=y_{n_0}(s)\in k_0$ for all $s\leqslant 0$. Moreover, $z(n)$ is a solution of

$$x(n+1)=g(n,x(n))-\sum_{s=-\infty}^{0}G(n,s,x(n+s),x(n))+q(n)$$

where $q(n)=f(n+n_m,z(n))-\sum_{s=-\infty}^{0}F(n+n_m,s,z(n+s),z(n))+h(n+n_m)-g(n,z(n))-\sum_{s=-\infty}^{0}G(n,s,z(n+s),z(n))$.

Since $|z(n)|\leqslant c$ for $n\leqslant\delta-n_m$ and small $\varepsilon>0$ and since $|h(n+n_m)|<\delta(\varepsilon/2)/2$ for $n\geqslant n_0-n_m$, we have $|q(n)|<\delta(\varepsilon/2)$ for $n_0-n_m\leqslant n\leqslant\delta-n_m$ by (7). Since $n_0\geqslant n_m$ and $\rho(v_{n_0-n_m},u_{n_0})<\delta(\varepsilon/2)/2$ by (8) and $\rho(u_{n_0},z_{n_0-n_m})=\rho(u_{n_0},y_{n_0})<\delta(\varepsilon/2)/2$, we have $\rho(v_{n_0-n_m},z_{n_0-n_m})\leqslant\rho(v_{n_0-n_m},u_{n_0})+\rho(u_{n_0},z_{n_0-n_m})<\delta(\varepsilon/2)$.

Thus the total stability of $v(n)$ implies that $\rho(v_{\delta-n_m},z_{\delta-n_m})<\varepsilon/2$.

On the other hand, Eq. (8) implies that $\rho(u_n,v_{n-n_m})<\delta(\varepsilon/2)/2$ for $n\geqslant n_0$.

Therefore, if $n_0\geqslant\alpha(\varepsilon)$, $\rho(u_{n_0},y_{n_0})<\delta(\varepsilon/2)/2$ and $|h(n)|<\delta(\varepsilon/2)/2$ for $n\geqslant n_0$, then we have $\rho(u_\delta,y_\delta)\leqslant\rho(u_\delta,v_{\delta-t_m})+\rho(v_{\delta-t_m},z_{\delta-t_m})<\varepsilon$. This contradicts $\rho(u_\delta,y_\delta)=\varepsilon$. Thus $\rho(u_n,y_n)<\varepsilon$ for all $n\geqslant n_0$, if $n_0\geqslant\alpha(\varepsilon)$, $\rho(u_{n_0},y_{n_0})<\delta^*(\varepsilon)$ and $|h(n)|<\delta^*(\varepsilon)$ for all $n\geqslant n_0$, where $\delta^*(\varepsilon)=\delta(\varepsilon/2)/2$. This shows that $u(n)$ is eventually totally (k_0,ρ) -stable.

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Volterra 差分方程的渐近概周期解的存在和稳定性

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摘　要:对于具有无界时滞的非线性 Volterra 差分方程,通过有界解的某种完全稳定性刻画了解的完全稳定性和渐近概周期性的存在.

关键词: Volterra 差分方程;渐近概周期解;完全稳定性

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