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三基因相互作用调控模型的 hopf 分支

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摘　要:以 $\tau_1+\tau_2+\tau_3$ 为参数,得到正平衡点的稳定性以及 Hopf 分支的存在性,并使用规范型和中心流形定理,获得了 Hopf 分支的方向和分支周期解稳定性的计算公式.

关键词:时滞;正平衡点;稳定性;Hopf 分支
中图分类号:O175　　**文献标志码**: A

在文献[1]中,作者建立了如下的基因调控模型:

$$\begin{cases}\dfrac{dx_1}{dt}=\dfrac{a_1}{1+k_1x_3^q(t-\tau_1)}-d_1x_1\\ \dfrac{dx_2}{dt}=\dfrac{1+k_2x_1^q(t-\tau_2)}{k+k_2x_1^q(t-\tau_2)}-d_2x_2\\ \dfrac{dx_3}{dt}=\dfrac{a_3}{1+k_3x_2^q(t-\tau_3)}-d_3x_3\end{cases}\tag{1}$$

其中 $x_1(t),x_2(t),x_3(t)$ 分别表示基因的浓度,$a_1,a_3,k_1,k_2,k_3,d_1,d_2,d_3$ 均为正常数,$k>1$(为整数),$q\geqslant1$ 为 Hill 常数且为整数,$\tau_i(i=1,2,3)$为时滞.

1　正平衡点的稳定性和局部 Hopf 分支的存在性

设 $E^*(x_1^*,x_2^*,x_3^*)$是系统(1)的正平衡点,则系统(1)在正平衡点 $E^*(x_1^*,x_2^*,x_3^*)$处的线性近似系统为:

$$\begin{cases}\dfrac{dx_1}{dt}=-d_1x_1+a_{11}x_3(t-\tau_1)\\ \dfrac{dx_2}{dt}=-d_2x_2+a_{22}x_1(t-\tau_2)\\ \dfrac{dx_3}{dt}=-d_3x_3+a_{33}x_2(t-\tau_3)\end{cases}\tag{2}$$

其中 $a_{11}=-\dfrac{a_1k_1q(x_3^*)^{q-1}}{[1+k_1(x_3^*)^q]^2}<0,a_{22}=\dfrac{(k-1)k_2q(x_1^*)^{q-1}}{[k+k_2(x_1^*)^q]^2}>0,a_{33}=-\dfrac{a_3k_3q(x_2^*)^{q-1}}{[1+k_3(x_2^*)^q]^2}<0.$

引理 1[2]　对于方程 $h(z)=z^3+az^2+bz+c,c>0$,记 z_1^*,z_2^* 分别为 $h'(z)=3z^2+2az+b=0$ 的两个实根,且 $z_1^*<z_2^*$,则:①如果 $a^2-3b\leqslant0$,则 $h(z)=0$ 无正解;②如果 $a^2-3b>0$,且 $b>0,a>0$,则 z_1^*,z_2^* 均小于零,$h(z)=0$ 无正解;③如果 $a^2-3b>0$,且 $b>0,a<0$,则 z_1^*,z_2^* 均大于零,$h(z)=0$ 可能有正解;④如果 $a^2-3b>0$,且 $b>0$,则 $z_1^*z_2^*<0,h(z)=0$ 可能有正解;⑤当 $h(z)=0$ 可能有正解时,如果条件 $2a^3-9ab-2(a^2-$

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$3b)^{\frac{3}{2}}+27c<0$ 成立,则 $h(z)=0$ 一定有两个正解.

关于 E^* 的特征方程为:

$$(\lambda+d_1)(\lambda+d_2)(\lambda+d_3)-a_{11}a_{22}a_{33}e^{-\lambda(\tau_1+\tau_2+\tau_3)}=0 \tag{3}$$

记 $\tau=\tau_1+\tau_2+\tau_3$,当 $\tau=0$ 时,式(3)可简化为:

$$\lambda^3+A_1\lambda^2+A_2\lambda+A_3=0 \tag{4}$$

其中 $A_1=d_1+d_2+d_3>0, A_2=d_1d_2+d_1d_3+d_2d_3>0, A_3=d_1d_2d_3-a_{11}a_{22}a_{33}>0$.

由 Routh-Hurwitz 条件易知,方程(4)的所有根均为负实部,也就是说当 $\tau=0$ 时,E^* 为局部渐近稳定的. 方程(3)可化为:

$$\lambda^3+A_1\lambda^2+A_2\lambda+A_4+A_5e^{-\lambda(\tau_1+\tau_2+\tau_3)}=0 \tag{5}$$

其中 $A_4=d_1d_2d_3, A_5=-a_{11}a_{22}a_{33}$. 当 $\tau\neq0$ 时,设 $\lambda=i\omega(\omega>0)$ 为方程(5)的一个根,则代入式(5)可得:

$$-i\omega^3-A_1\omega^2+iA_2\omega+A_4+A_5(\cos\omega\tau-i\sin\omega\tau)=0$$

令 $z=\omega^2$,则化为:

$$z^3+(A_1-2A_2)z^2+(A_2^2-2A_1A_4)z+A_4^2-A_5^2=0 \tag{6}$$

由 A_4,A_5 的表达式知:$A_4^2-A_5^2>0$,令式(6)中 $A_i(i=1,2,3,4,5)$ 满足引理 1 中有正解的条件,则方程有两个正根,不妨设为 z_1,z_2,则 $\omega_1=\sqrt{z_1}, \omega_2=\sqrt{z_2}$,则方程(5)有两对纯虚根 $\pm i\omega_k, k=1,2$. 虚部与实部分离得: $\tan(\omega_k\tau)=\dfrac{A_2\omega-\omega^3}{A_1\omega^2-A_4}$. 则 $\tau_{kn}=\dfrac{1}{\omega_k}\arctan\left\{\dfrac{A_2\omega-\omega^3}{A_1\omega^2-A_4}\right\}+\dfrac{n\pi}{\omega_k}; k=1,2; n=0,1,2,\cdots$.

引理 2　横截条件 $\dfrac{d\mathrm{Re}(\lambda)}{d\tau}\mid\tau=\tau_0>0$.

综上所述,根据文献[3,4]可得如下定理:

定理 1　(i)当 $A_i(i=1,2,3,4,5)$ 满足引理 1 中①或②时,方程(5)无正解,E^* 为绝对稳定的平衡点,其稳定性与 τ 的取值无关;(ii)当 $A_i(i=1,2,3,4,5)$ 满足引理 1 中③⑤或④⑤时,方程(5)有正解,且当 ω_0 满足 $\omega_0^2>A_2$ 时,有:$\tau\in[0,\tau_0)$ 时,E^* 为局部渐近稳定的;当 $\tau=\tau_0$ 时,系统在 E^* 处产生 Hopf 分支.

2　Hopf 分支的方向和分支周期解的稳定性

在这一部分,通过使用泛函微分方程的规范型理论和中心流行定理[5],研究 Hopf 分支的方向和分支周期解的稳定性的计算公式.

令 $\tau=\tau_{kn}+\mu$,则 $\mu=0$ 是系统(1)的 Hopf 分支值,$\mu\in R$. 令 $u_1(t)=x_1(t)-x_1^*, u_2(t)=x_2(t)-x_2^*, u_3(t)=x_3(t)-x_3^*$. 则系统(1)可写成:

$$\begin{cases} u'_1(t)=-d_1u_1(t)+a_{11}u_3(t-\tau_1)+\sum\limits_{i+j\geqslant2}\dfrac{1}{i!j!}f_{ij}^{(1)}u_1^i(t)u_3^j(t-\tau_1) \\ u'_2(t)=-d_2u_2(t)+a_{22}u_1(t-\tau_2)+\sum\limits_{i+j\geqslant2}\dfrac{1}{i!j!}f_{ij}^{(2)}u_1^i(t-\tau_2)u_2^j(t) \\ u'_3(t)=-d_3u_3(t)+a_{33}u_2(t-\tau_3)+\sum\limits_{i+j\geqslant2}\dfrac{1}{i!j!}f_{ij}^{(3)}u_2^i(t-\tau_3)u_3^j(t) \end{cases} \tag{7}$$

为简单起见,将式(6)记作:

$$u'(t)=L_\mu(u_t)+f(\mu,u_t) \tag{8}$$

根据 Riesz 表示定理,存在有界变差矩阵 $\eta(\theta,\mu), \theta\in[-\tau_1,0]$,使 $L_\mu\varphi=\int_{-\tau_1}^0\varphi(\theta)d\eta(\theta,0)$.

对于 $\varphi\in C^1([-\tau_1,0],R^3)$,定义:

$$A(\mu)\varphi=\begin{cases}\dfrac{d\varphi(\theta)}{d\theta}, \theta\in[-\tau_1,0) \\ \int_{-\tau_1}^0 d\eta(\mu,s)\varphi(s), \theta=0\end{cases}, \qquad R(\mu)\varphi=\begin{cases}0, \theta\in[-\tau_1,0) \\ f(\mu,\varphi), \theta=0\end{cases}$$

则式(8)等价于:

$$u'(t) = A(\mu)u_t + R(\mu)u_t \tag{9}$$

对于 $\psi \in C^1([0,\tau_1],R^3)$,定义 $A^*\psi(s) = \begin{cases} -\dfrac{d\psi(s)}{ds}, s \in (0,\tau_1] \\ \int_{-\tau_1}^{0} d\eta(t,0)\psi(-t), s=0 \end{cases}$.

设 $q(\theta)=(1,\alpha,\beta)^{T}e^{i\omega_0\theta}$是对应于 $i\omega_0$ 的特征向量,则 $q(0)=\left(1,\dfrac{a_{22}e^{-i\omega_0\tau_2}}{i\omega_0+d_2},\dfrac{i\omega_0+d_1}{a_{11}e^{-i\omega_0\tau_1}}\right)^{T}$,同理得 A^* 的特征向量 $q^*(s)=D(1,\sigma,\rho)e^{i\omega_0 s}=D\left(1,\dfrac{-d_1+i\omega_0}{a_{22}e^{-i\omega_0\tau_2}},\dfrac{a_{11}e^{-i\omega_0\tau_1}}{-d_3+i\omega_0}\right)e^{i\omega_0 s}$. 为了使 $<q^*(s),q(\theta)>=1$,取 $\bar{D}=(1+\alpha\bar{\sigma}+\beta\bar{\rho}+\beta a_{11}\tau_2 e^{-i\omega_0\tau_1}+\bar{\sigma}a_{22}\tau_2 e^{-i\omega_0\tau_2}+\bar{\rho}\alpha a_{33}\tau_3 e^{-i\omega_0\tau_3})^{-1}$.

下面,首先计算在 $\mu=0$ 处的中心流行 C_0,在 $\mu=0$ 时,令 u_t 是方程(9)的解并定义 $z(t)=<q^*,u_t>$,$W(t,\theta)=u_t(\theta)-z(t)q(\theta)-\overline{z(t)q(\theta)}=u_t(\theta)-2\mathrm{Re}\{z(t)q(\theta)\}$.

而 $W(t,\theta)=W_{20}(\theta)\dfrac{z^2}{2}+W_{11}(\theta)z\bar{z}+W_{02}(\theta)\dfrac{\bar{z}^2}{2}+\cdots$.

事实上,$z(t)$,$\overline{z(t)}$分别是在 q 和 q^* 方向上中心流行的局部坐标. 由于解 $u_t\in C_0$,于是得到:$z'(t)=<q^*,u'_t>=<q^*,Au_t+Ru_t>=<q^*,Au_t>+<q^*,Ru_t>=i\omega_0 z+\overline{q^*}(0)f(0,W(z,\bar{z},0)+2\mathrm{Re}\{zq(0)\})=i\omega_0 z+\overline{q^*}(0)f_0(z,\bar{z})$.

另记 $z'(t)=i\omega_0 z+g(z,\bar{z})$,其中 $g(z,\bar{z})=g_{20}\dfrac{z^2}{2}+g_{11}z\bar{z}+g_{02}\dfrac{\bar{z}^2}{2}+\cdots$.

$$W' = u'_t - z'q - \bar{z}'\bar{q} = \begin{cases} AW - 2\mathrm{Re}[\overline{q^*}(0)f_0(z,\bar{z})q(\theta)], \theta \in [-\tau_1,0) \\ AW - 2\mathrm{Re}[\overline{q^*}(0)f_0(z,\bar{z})q(\theta)] + f_0(z,\bar{z}), \theta = 0 \end{cases} \tag{10}$$

令 $W'=AW+H(z,\bar{z},\theta)$,又 $W'=W_z z'+W_{\bar{z}}\bar{z}'$. $H(z,\bar{z},\theta)=H_{20}\dfrac{z^2}{2}+H_{11}z\bar{z}+H_{02}\dfrac{\bar{z}^2}{2}+\cdots$,可解得:

$$(A-2i\omega_0)W_{20}(\theta) = -H_{20}(\theta) \tag{11}$$

$$AW_{11}(\theta) = -H_{11}(\theta) \tag{12}$$

由 $u_t(\theta)=(u_{1t}(\theta),u_{2t}(\theta),u_{3t}(\theta))=W(t,\theta)+z(t)q(\theta)+\overline{z(t)q(\theta)}$,$q(\theta)=(1,\alpha,\beta)^{T}e^{i\omega_0\theta}$得:

$$\begin{aligned}
g(z,\bar{z}) = {} & \bar{D}\{[\tfrac{1}{2}f_{20}^{(1)}+\beta e^{-i\omega_0\tau_1}f_{11}^{(1)}+\tfrac{1}{2}\beta^2 e^{-2i\omega_0\tau_1}f_{02}^{(1)}]z^2+[f_{20}^{(1)}+f_{11}^{(1)}(\beta e^{-i\omega_0\tau_1}+\bar{\beta}e^{i\omega_0\tau_1})+ \\
& \beta\bar{\beta}f_{02}^{(1)}]z\bar{z}+[\tfrac{1}{2}f_{20}^{(1)}+\bar{\beta}e^{i\omega_0\tau_1}f_{11}^{(1)}+\tfrac{1}{2}\bar{\beta}^2 e^{2i\omega_0\tau_1}f_{02}^{(1)}]\bar{z}^2\}+\bar{D}\bar{\sigma}\{[\tfrac{1}{2}e^{-2i\omega_0\tau_2}f_{20}^{(2)}+\alpha e^{-i\omega_0\tau_2}f_{11}^{(2)}+ \\
& \tfrac{1}{2}\alpha^2 f_{02}^{(2)}]z^2+[f_{20}^{(2)}+f_{11}^{(2)}(\alpha e^{i\omega_0\tau_2}+\bar{\alpha}e^{-i\omega_0\tau_2})+\alpha\bar{\alpha}f_{02}^{(2)}]z\bar{z}+[\tfrac{1}{2}e^{2i\omega_0\tau_2}f_{20}^{(2)}+\bar{\alpha}e^{i\omega_0\tau_2}f_{11}^{(2)}+ \\
& \tfrac{1}{2}\bar{\alpha}^2 f_{02}^{(2)}]\bar{z}^2\}+\bar{D}\bar{\rho}\{[\tfrac{1}{2}\alpha^2 e^{-2i\omega_0\tau_3}f_{20}^{(3)}+\alpha\beta e^{-i\omega_0\tau_3}f_{11}^{(3)}+\tfrac{1}{2}\beta^2 f_{02}^{(3)}]z^2+[\alpha\bar{\alpha}f_{20}^{(3)}+f_{11}^{(3)}(\alpha\bar{\beta}e^{-i\omega_0\tau_3}+ \\
& \bar{\alpha}\beta e^{i\omega_0\tau_3})+\beta\bar{\beta}f_{02}^{(3)}]z\bar{z}+[\tfrac{1}{2}\bar{\alpha}^2 e^{2i\omega_0\tau_3}f_{20}^{(3)}+\bar{\alpha}\bar{\beta}e^{i\omega_0\tau_3}f_{11}^{(3)}+\tfrac{1}{2}\bar{\beta}^2 f_{02}^{(3)}]\bar{z}^2\}+\bar{D}\{\tfrac{1}{2}f_{20}^{(1)}[W_{20}^{(1)}(0)+ \\
& 2W_{11}^{(1)}(0)]+f_{11}^{(1)}[\tfrac{1}{2}\bar{\beta}e^{i\omega_0\tau_1}W_{20}^{(1)}(0)+\tfrac{1}{2}W_{20}^{(3)}(-\tau_1)+W_{11}^{(3)}(-\tau_1)+\beta e^{-i\omega_0\tau_1}W_{11}^{(1)}(0)]+ \\
& \tfrac{1}{2}f_{02}^{(1)}[\bar{\beta}e^{i\omega_0\tau_1}W_{20}^{(3)}(-\tau_1)+2\beta e^{-i\omega_0\tau_1}W_{11}^{(3)}(-\tau_1)]\}z^2\bar{z}+\bar{D}\bar{\sigma}\{\tfrac{1}{2}f_{20}^{(2)}[e^{i\omega_0\tau_2}W_{20}^{(1)}(-\tau_2)+ \\
& 2e^{i\omega_0\tau_2}W_{11}^{(1)}(-\tau_2)]+f_{11}^{(2)}[\tfrac{1}{2}\bar{\alpha}W_{20}^{(1)}(-\tau_2)+\alpha W_{11}^{(1)}(-\tau_2)+\tfrac{1}{2}e^{i\omega_0\tau_2}W_{20}^{(2)}(0)+e^{-i\omega_0\tau_2}W_{11}^{(2)}(0)+ \\
& \tfrac{1}{2}f_{02}^{(2)}[\bar{\alpha}W_{20}^{(2)}(0)+2\alpha W_{11}^{(2)}(0)]\}z^2\bar{z}+\bar{D}\bar{\rho}\{\tfrac{1}{2}f_{20}^{(3)}[\bar{\alpha}e^{i\omega_0\tau_3}W_{20}^{(2)}(-\tau_3)+2\alpha e^{-i\omega_0\tau_3}W_{11}^{(2)}(-\tau_3)]f_{11}^{(3)} \\
& [\tfrac{1}{2}\bar{\beta}W_{20}^{(2)}(-\tau_3)+\tfrac{1}{2}\bar{\alpha}e^{i\omega_0\tau_3}W_{20}^{(3)}(0)+\beta W_{11}^{(2)}(-\tau_3)+\alpha e^{-i\omega_0\tau_3}W_{11}^{(3)}(0)+\tfrac{1}{2}f_{02}^{(3)}[\bar{\beta}W_{20}^{(3)}(0)+
\end{aligned}$$

$$2\beta W_{11}^{(3)}(0)]\}z^2\bar{z}+\bar{D}[\frac{1}{2}f_{12}^{(1)}(\beta^2 e^{-2i\omega_0\tau_1}+2\beta\bar{\beta})+\frac{1}{2}f_{21}^{(1)}(\bar{\beta}e^{i\omega_0\tau_1}+2\beta e^{-i\omega_0\tau_1})+\frac{1}{2}f_{30}^{(1)}+$$
$$\frac{1}{2}\beta^2\bar{\beta}e^{-i\omega_0\tau_1}f_{03}^{(1)}]z^2\bar{z}+\bar{D}\bar{\sigma}[\frac{1}{2}f_{12}^{(2)}(\alpha^2 e^{i\omega_0\tau_2}+2\alpha\bar{\alpha}e^{-i\omega_0\tau_2})+\frac{1}{2}f_{21}^{(2)}(\bar{\alpha}e^{-2i\omega_0\tau_2}+2\alpha)+$$
$$\frac{1}{2}e^{-i\omega_0\tau_2}f_{30}^{(2)}+\frac{1}{2}\alpha^2\bar{\alpha}f_{03}^{(2)}]z^2\bar{z}+\bar{D}\bar{\rho}[\frac{1}{2}f_{12}^{(3)}(\bar{\alpha}\beta^2 e^{i\omega_0\tau_3}+2\alpha\beta\bar{\beta}e^{-i\omega_0\tau_3})+\frac{1}{2}f_{21}^{(3)}(\alpha^2\bar{\beta}e^{-2i\omega_0\tau_3}+$$
$$2\alpha\bar{\alpha}\beta)+\frac{1}{2}\alpha^2\bar{\alpha}e^{-i\omega_0\tau_3}f_{30}^{(3)}+\frac{1}{2}\beta^2\bar{\beta}f_{03}^{(3)}]z^2\bar{z}+\cdots$$

比较系数得:

$$g_{20}=\bar{D}[f_{20}^{(1)}+2f_{11}^{(1)}\beta e^{-i\omega_0\tau_1}+f_{02}^{(1)}\beta^2 e^{-2i\omega_0\tau_1}+\bar{\sigma}(f_{20}^{(2)}e^{-2i\omega_0\tau_2}+2f_{11}^{(2)}\alpha e^{-i\omega_0\tau_2}+$$
$$f_{02}^{(2)}\alpha^2)+\bar{\rho}(f_{20}^{(3)}\alpha^2 e^{-2i\omega_0\tau_3}+2f_{11}^{(3)}\alpha\beta e^{-i\omega_0\tau_3}+f_{02}^{(3)}\beta^2)]$$
$$g_{11}=\bar{D}\{f_{20}^{(1)}+f_{11}^{(1)}(\beta e^{-i\omega_0\tau_1}+\bar{\beta}e^{i\omega_0\tau_1})+f_{02}^{(1)}\beta\bar{\beta}+\bar{\sigma}[f_{20}^{(2)}+f_{11}^{(2)}(\alpha e^{i\omega_0\tau_2}+\bar{\alpha}e^{-i\omega_0\tau_2})+$$
$$f_{02}^{(2)}\alpha\bar{\alpha}]+\bar{\rho}[f_{20}^{(3)}\alpha\bar{\alpha}+f_{11}^{(3)}(\alpha\bar{\beta}e^{-i\omega_0\tau_3}+\bar{\alpha}\beta e^{i\omega_0\tau_3})+\beta\bar{\beta}f_{02}^{(3)}]\}$$
$$g_{02}=\bar{D}[f_{20}^{(1)}+2f_{11}^{(1)}\bar{\beta}e^{i\omega_0\tau_1}+f_{02}^{(1)}\bar{\beta}^2 e^{2i\omega_0\tau_1}+\bar{\sigma}(f_{20}^{(2)}e^{2i\omega_0\tau_2}+2f_{11}^{(2)}\bar{\alpha}e^{i\omega_0\tau_2}+$$
$$\bar{\alpha}^2 f_{02}^{(2)})+\bar{\rho}(f_{20}^{(3)}\bar{\alpha}^2 e^{2i\omega_0\tau_3}+2f_{11}^{(3)}\bar{\alpha}\bar{\beta}e^{i\omega_0\tau_3}+\bar{\beta}^2 f_{02}^{(3)})]$$
$$g_{21}=\bar{D}\{f_{20}^{(1)}(W_{20}^{(1)}(0)+2W_{11}^{(1)}(0))+f_{11}^{(1)}[\bar{\beta}e^{i\omega_0\tau_1}W_{20}^{(1)}(0)+W_{20}^{(3)}(-\tau_1)+2W_{11}^{(3)}(-\tau_1)+$$
$$2\beta e^{-i\omega_0\tau_1}W_{11}^{(1)}(0)]+f_{02}^{(1)}[\bar{\beta}e^{i\omega_0\tau_1}W_{20}^{(3)}(-\tau_1)+2\beta e^{-i\omega_0\tau_1}W_{11}^{(3)}(-\tau_1)]+f_{12}^{(1)}[\beta^2 e^{-2i\omega_0\tau_1}+2\beta\bar{\beta}]+$$
$$f_{21}^{(1)}[\bar{\beta}e^{i\omega_0\tau_1}+2\beta e^{-i\omega_0\tau_1}]+f_{30}^{(1)}+f_{03}^{(1)}\beta^2\bar{\beta}e^{-i\omega_0\tau_1}\}+\bar{D}\bar{\sigma}\{f_{20}^{(2)}(e^{i\omega_0\tau_2}W_{20}^{(1)}(-\tau_2)+$$
$$2e^{i\omega_0\tau_2}W_{11}^{(1)}(-\tau_2))+f_{11}^{(2)}[\bar{\alpha}W_{20}^{(1)}(-\tau_2)+2\alpha W_{11}^{(1)}(-\tau_2)+e^{i\omega_0\tau_2}W_{20}^{(2)}(0)+2e^{-i\omega_0\tau_2}W_{11}^{(2)}(0)]+$$
$$f_{02}^{(2)}[\bar{\alpha}W_{20}^{(2)}(0)+2\alpha W_{11}^{(2)}(0)]+f_{12}^{(2)}[\alpha^2 e^{i\omega_0\tau_2}+2\alpha\bar{\alpha}e^{-i\omega_0\tau_2}]+f_{21}^{(2)}[\bar{\alpha}e^{-2i\omega_0\tau_2}+2\alpha]+f_{30}^{(2)}e^{-i\omega_0\tau_2}+$$
$$f_{03}^{(2)}\alpha^2\bar{\alpha}\}+\bar{D}\bar{\rho}\{f_{20}^{(3)}[\bar{\alpha}e^{i\omega_0\tau_3}W_{20}^{(2)}(-\tau_3)+2\alpha e^{-i\omega_0\tau_3}W_{11}^{(2)}(-\tau_3)]+f_{11}^{(3)}[\bar{\beta}W_{20}^{(2)}(-\tau_3)+$$
$$2\beta W_{11}^{(2)}(-\tau_3)+\bar{\alpha}e^{i\omega_0\tau_3}W_{20}^{(3)}(0)+2\alpha e^{-i\omega_0\tau_3}W_{11}^{(3)}(0)]+f_{02}^{(3)}[\bar{\beta}W_{20}^{(3)}(0)+2\beta W_{11}^{(3)}(0)]+$$
$$f_{12}^{(3)}[\bar{\alpha}\beta^2 e^{i\omega_0\tau_3}+2\alpha\beta\bar{\beta}e^{-i\omega_0\tau_3}]+f_{21}^{(3)}[\alpha^2\bar{\beta}e^{-2i\omega_0\tau_3}+2\alpha\bar{\alpha}\beta]+f_{30}^{(3)}\alpha^2\bar{\alpha}e^{-i\omega_0\tau_3}+f_{03}^{(3)}\beta^2\bar{\beta}\}$$

把式(10)代入式(11)得到 $W'_{20}(\theta)=2i\omega_0 W_{20}(\theta)+g_{20}q(\theta)+\overline{g_{02}}\,\overline{q(\theta)}$. 解得 $W_{20}(\theta)=\frac{ig_{20}}{\omega_0}q(0)e^{i\omega_0\theta}+\frac{i\overline{g_{02}}}{3\omega_0}\overline{q(0)}e^{-i\omega_0\theta}+E_1 e^{2i\omega_0\theta}$,同理得 $W_{11}(\theta)=-\frac{ig_{11}}{\omega_0}q(0)e^{i\omega_0\theta}+\frac{i\overline{g_{11}}}{\omega_0}\overline{q(0)}e^{-i\omega_0\theta}+E_2$.

由 A 的定义得:

$$\int_{-\tau_1}^{0}d\eta(\theta)W_{20}(\theta)=2i\omega_0 W_{20}(0)-H_{20}(0)$$
$$\int_{-\tau_1}^{0}d\eta(\theta)W_{11}(\theta)=-H_{11}(0)$$
$$H_{20}(0)=-g_{20}q(0)-\overline{g_{02}}\,\overline{q(0)}+\begin{pmatrix}f_{20}^{(1)}+2\beta e^{-i\omega_0\tau_1}f_{11}^{(1)}+\beta^2 e^{-2i\omega_0\tau_1}f_{02}^{(1)}\\ f_{20}^{(2)}e^{-2i\omega_0\tau_2}+2\alpha e^{-i\omega_0\tau_2}f_{11}^{(2)}+\alpha^2 f_{02}^{(2)}\\ \alpha^2 e^{-2i\omega_0\tau_3}f_{20}^{(3)}+2\alpha\beta e^{-i\omega_0\tau_3}f_{11}^{(3)}+\beta^2 f_{02}^{(3)}\end{pmatrix}$$
$$[2i\omega_0 I-\int_{-\tau_1}^{0}e^{2i\omega_0\theta}d\eta(\theta)]E_1=\begin{pmatrix}f_{20}^{(1)}+2f_{11}^{(1)}\beta e^{-i\omega_0\tau_1}+f_{02}^{(1)}\beta^2 e^{-2i\omega_0\tau_1}\\ f_{20}^{(2)}e^{-2i\omega_0\tau_2}+2f_{11}^{(2)}\alpha e^{-i\omega_0\tau_2}+f_{02}^{(2)}\alpha^2\\ f_{20}^{(3)}\alpha^2 e^{-2i\omega_0\tau_3}+2f_{11}^{(3)}\alpha\beta e^{-i\omega_0\tau_3}+f_{02}^{(3)}\beta^2\end{pmatrix}$$
$$E_1^{(1)}=\frac{1}{A}\begin{vmatrix}f_{20}^{(1)}+2f_{11}^{(1)}\beta e^{-i\omega_0\tau_1}+f_{02}^{(1)}\beta^2 e^{2i\omega_0\tau_1} & 0 & -a_{11}e^{-2i\omega_0\tau_1}\\ f_{20}^{(2)}e^{-2i\omega_0\tau_2}+2f_{11}^{(2)}\alpha e^{-i\omega_0\tau_2}+f_{02}^{(3)}\beta^2 & 2i\omega_0+d_2 & 0\\ f_{20}^{(3)}\alpha^2 e^{-2i\omega_0\tau_3}+2f_{11}^{(3)}\alpha\beta e^{-i\omega_0\tau_3}+f_{02}^{(3)}\beta^2 & -a_{33}e^{-2i\omega_0\tau_3} & 2i\omega_0+d_3\end{vmatrix}$$

$$E_1^{(2)}=\frac{1}{A}\begin{vmatrix}2i\omega_0+d_1 & f_{20}^{(1)}+2f_{11}^{(1)}\beta e^{-i\omega_0\tau_1}+f_{02}^{(1)}\beta^2 e^{2i\omega_0\tau_1} & -a_{11}e^{-2i\omega_0\tau_1}\\ -a_{22}e^{-2i\omega_0\tau_2} & f_{20}^{(2)}e^{-2i\omega_0\tau_2}+2f_{11}^{(2)}\alpha e^{-i\omega_0\tau_2}+f_{02}^{(3)}\beta^2 & 0\\ 0 & f_{20}^{(3)}\alpha^2 e^{-2i\omega_0\tau_3}+2f_{11}^{(3)}\alpha\beta e^{-i\omega_0\tau_3}+f_{02}^{(3)}\beta^2 & 2i\omega_0+d_3\end{vmatrix}$$

$$E_1^{(3)}=\frac{1}{A}\begin{vmatrix}2i\omega_0+d_1 & 0 & f_{20}^{(1)}+2f_{11}^{(1)}\beta e^{-i\omega_0\tau_1}+f_{02}^{(1)}\beta^2 e^{2i\omega_0\tau_1}\\ -a_{22}e^{-2i\omega_0\tau_2} & 2i\omega_0+d_2 & f_{20}^{(2)}e^{-2i\omega_0\tau_2}+2f_{11}^{(2)}\alpha e^{-i\omega_0\tau_2}+f_{02}^{(3)}\beta^2\\ 0 & -a_{33}e^{-2i\omega_0\tau_3} & f_{20}^{(3)}\alpha^2 e^{-2i\omega_0\tau_3}+2f_{11}^{(3)}\alpha\beta e^{-i\omega_0\tau_3}+f_{02}^{(3)}\beta^2\end{vmatrix}$$

同理可得:

$$E_2^{(1)}=\frac{1}{B}\begin{vmatrix}f_{20}^{(1)}+f_{11}^{(1)}(\beta e^{-i\omega_0\tau_1}+\bar\beta e^{i\omega_0\tau_1})+f_{02}^{(1)}\beta\bar\beta & 0 & -a_{11}\\ f_{20}^{(2)}+f_{11}^{(2)}(\alpha e^{i\omega_0\tau_2}+\bar\alpha e^{-i\omega_0\tau_2})+f_{02}^{(2)}\alpha\bar\alpha & d_2 & 0\\ f_{20}^{(3)}\alpha\bar\alpha+f_{11}^{(3)}(\alpha\bar\beta e^{-i\omega_0\tau_3}+\bar\alpha\beta e^{i\omega_0\tau_3})+f_{02}^{(3)}\beta\bar\beta & -a_{33} & d_3\end{vmatrix}$$

$$E_2^{(2)}=\frac{1}{B}\begin{vmatrix}d_1 & f_{20}^{(1)}+f_{11}^{(1)}(\beta e^{-i\omega_0\tau_1}+\bar\beta e^{i\omega_0\tau_1})+f_{02}^{(1)}\beta\bar\beta & -a_{11}\\ -a_{22} & f_{20}^{(2)}+f_{11}^{(2)}(\alpha e^{i\omega_0\tau_2}+\bar\alpha e^{-i\omega_0\tau_2})+f_{02}^{(2)}\alpha\bar\alpha & 0\\ 0 & f_{20}^{(3)}\alpha\bar\alpha+f_{11}^{(3)}(\alpha\bar\beta e^{-i\omega_0\tau_3}+\bar\alpha\beta e^{i\omega_0\tau_3})+f_{02}^{(3)}\beta\bar\beta & d_3\end{vmatrix}$$

$$E_2^{(3)}=\frac{1}{B}\begin{vmatrix}d_1 & 0 & f_{20}^{(1)}+f_{11}^{(1)}(\beta e^{-i\omega_0\tau_1}+\bar\beta e^{i\omega_0\tau_1})+f_{02}^{(1)}\beta\bar\beta\\ -a_{22} & d_2 & f_{20}^{(2)}+f_{11}^{(2)}(\alpha e^{i\omega_0\tau_2}+\bar\alpha e^{-i\omega_0\tau_2})+f_{02}^{(2)}\alpha\bar\alpha\\ 0 & -a_{33} & f_{20}^{(3)}\alpha\bar\alpha+f_{11}^{(3)}(\alpha\bar\beta e^{-i\omega_0\tau_3}+\bar\alpha\beta e^{i\omega_0\tau_3})+f_{02}^{(3)}\beta\bar\beta\end{vmatrix}$$

其中 $A=\begin{vmatrix}2i\omega_0+d_1 & 0 & -a_{11}e^{-2i\omega_0\tau_1}\\ -a_{22}e^{-2i\omega_0\tau_2} & 2i\omega_0+d_2 & 0\\ 0 & -a_{33}e^{-2i\omega_0\tau_3} & 2i\omega_0+d_3\end{vmatrix}$, $B=\begin{vmatrix}d_1 & 0 & -a_{11}\\ -a_{22} & d_2 & 0\\ 0 & -a_{33} & d_3\end{vmatrix}$.

这样,g_{21} 就能用参数和时滞确定. 于是得到 $C_1(0)=\frac{i}{2\omega_0}\left(g_{20}g_{11}-2|g_{11}|^2-\frac{|g_{02}|^2}{3}\right)+\frac{g_{21}}{2}$, $\mu_2=-\frac{\mathrm{Re}\{C_1(0)\}}{\mathrm{Re}(\lambda'(\tau_k))}$, $\beta_2=2\mathrm{Re}\{C_1(0)\}$.

定理 2 (i) τ_{nk} 是系统(1)的 Hopf 分支值;(ii) μ_2 确定 Hopf 分支的方向:如果 $\mu_2>0$,则 Hopf 分支是超临界的,如果 $\mu_2<0$,则 Hopf 分支是次临界的;(iii) β_2 确定分支周期解的稳定性:若 $\beta_2<0$,则分支周期解是稳定的,若 $\beta_2>0$,则分支周期解是不稳定的.

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Hopf Bifurcation of Three Genes Regulatory Model for Interaction

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Abstract: By taking $\tau_1 + \tau_2 + \tau_3$ as parameters, this paper obtains the stability of positive equilibrium point and the existence of Hopf bifurcation, and uses normal form method and center manifold theorem to study an explicit algorithm for the direction of Hopf bifurcation and the stability of periodic solution of bifurcation.

Key words: time delay; positive equilibrium point; stability; Hopf bifurcation

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Properties of Compact Set in R^d

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Abstract: On defined distance ρ in K, that coK is the closed subspace of (K, ρ) is verified. Another distance ρ_2 is defined on coK and the same topology of ρ and ρ_2 on coK is induced.

Key words: compact set; convex compact set; support function

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