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关于拟正则密码群并半群的几个等式

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摘 要:在对拟正则密码群并半群的若干研究中,给出了拟正则密码群并半群、纯正拟正则密码群并半群及纯正的过阿贝尔的拟正则密码群并半群的等式刻画.

关键词:簇;拟正则密码群并半群;过阿贝尔的;纯正的
中图分类号: O152.7 **文献标识码**: A

令 S是完全正则半群 (也称为群并半群),称完全正则半群是纯正的,如果它的幂等元形成半群. 令 C是一个带簇,称纯正群并半群是 C纯正的,如果 $E(S) \in C$. 记纯正群并半群簇为 O,称完全正则半群是密码群并半群,如果它的 H关系是同余. 称密码群并半群 S是 C - 密码群并半群,如果 $S/H \in C$. 记密码群并半群为 BG.

令 S是完全正则半群,用 $L(CR)$表示完全正则半群簇的所有子簇,V表示完全正则半群簇的一个子簇. 称 $S \in HV$,如果对任意 $e \in E(S)$, $H_e \in V$. 特别地,一个完全正则半群 S是过阿贝尔的,若它是属于 HA,其中 A表示阿贝尔群簇.

在文献 [1]中给出了拟正则带簇的格图. 因为证明需要,在此把拟正则带簇的等式及表示记号给出.

$QRB = [axyzyxa = axyayzyayxa]$ (quasiregular bands).

定义 1 令 S是一密码群并半群,称 S是拟正则密码群并半群,如果 S/H是拟正则带,对应的完全正则半群簇记为 $QRBG$.

纯正密码群并半群是在完全正则半群研究中讨论地相对深入的半群,因此此处在拟正则密码群并半群的范围内,讨论了纯正拟正则密码群并半群及纯正的过阿贝尔的拟正则密码群并半群的等式刻画.

全文中 S表示完全正则半群,所有未曾解释的术语和符号在文献 [2][3]中均可找到.

为得到结论,先给出以下引理:

引理 1 令 $S = (Y; S_\alpha)$是纯正完全正则半群. 令 $\alpha, \beta \in Y$, $\alpha \leq \beta$,则 $E(S_\alpha)$是矩形带且对任意 $a, b \in S_\alpha$, $e \in E(S_\beta)$,有 $ab = aeb$.

引理 2 $O = [a^0 b^0 = (a^0 b^0)^0]$.

引理 3 $OHA = [ab = (a^0 b^0)^0 a^0 bab^0 (a^0 b^0)^0]$.

引理 4 $BG = [(ab)^0 = (a^0 b^0)^0] = [(a^2 xa^2)^0 = (axa)^0]$.

引理 5 对完全正则半群 S, 以下各款等价:

(i) S是 (完全)单半群; (ii) S满足等式 $(ab)^0 = (axb)^0$; (iii) S满足等式 $a^0 = (axa)^0$.

定理 1 $QRBG = [(axyzyxa)^0 = (axyayzyayxa)^0]$.

证明 令 S是拟正则密码群并半群. 由定义 1知, 对任意 $a, x, y, z \in S$, $(axyzyxa)\ H\ (axyayzyayxa)$. 所以 S满足等式 $(axyzyxa)^0 = (axyayzyayxa)^0$.

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另一方面,令 S 是一完全正则半群,且满足等式 $(axyzyxa)^0 = (axyayzyayxa)^0$,令 $a, b, x, y, z \in S$,替换 $x \to a^{-1}, y \to a^0, z \to aba$,则:

$$(aa^{-1}a^0(aba)a^0a^{-1}a)^0 = (aa^{-1}a^0aa^0(aba)a^0aa^0a^{-1}a)^0$$
$$(aba)^0 = (a^2ba^2)^0$$

由引理 4, $S \in BG$,再由 QRB 的等式,则 S 是拟正则密码群并半群.

定理 2 $OQRBG = [axyzyxa = (axyayzyayxa)^0 axyzyx^0y^0xa(axyayzyayxa)^0]$.

证明 令 $a, x, y, z \in S$,且 S 满足等式:

$$axyzyxa = (axyayzyayxa)^0 axyzyx^0y^0xa(axyayzyayxa)^0$$

则 $(axyzyxa)^0 = ((axyayzyayxa)^0 axyzyx^0y^0xa(axyayzyayxa)^0)^0 = (axyayzyayxa)^0$.

由定理 1知, $S \in QRBG$.

令 $e, f \in E(S)$, $a = (ef)^0$, $x = e$, $y = f$, $z = (ef)^0$. 则 S 满足的等式可化为:

等式左边 $= (ef)^0ef(ef)^0fe(ef)^0 = ef$;

等式右边 $= ((ef)^0ef(ef)^0f(ef)^0f(ef)^0fe(ef)^0)^0(ef)^0ef(ef)^0fefe(ef)^0((ef)^0ef(ef)^0f(ef)^0f(ef)^0fe(ef)^0)^0 = (ef)^0(ef)^0ef(ef)^0ef(ef)^0(ef)^0 = (ef)^2$.

所以,可以得到新等式: $ef = (ef)^2$,即有 $(ef)^0 = ef$,由引理 2, $S \in O$. 这样就证明了 $S \in OQRBG$.

另一方面,令 $S \in OQRBG$; $a, x, y, z \in S$. 由定理 1知, S 满足等式 $(axyzyxa)^0 = (axyayzyayxa)^0$. 则:

$axyzyxa = (axyzyxa)^0 axyzyxa(axyzyxa)^0 = (axyayzyayxa)^0 axyzyxa(axyayzyayxa)^0 =$
$(axyayzyayxa)^0 axyzyy^0x^0xa(axyayzyayxa)^0 = (axyayzyayxa)^0 axyzyy^0x^0y^0x^0xa(axyayzyayxa)^0 =$
$(axyayzyayxa)^0 axyzyx^0y^0xa(axyayzyayxa)^0$

由此可得: $OQRBG \subseteq [axyzyxa = (axyayzyayxa)^0 axyzyx^0y^0xa(axyayzyayxa)^0]$.

定理 3 $OQRBA = [axyzyxa = (axyayzyayxa)^0 axyzxya(axyayzyayxa)^0]$.

证明 令 $a, x, y, z \in S$,且 S 满足等式: $axyzyxa = (axyayzyayxa)^0 axyzxya(axyayzyayxa)^0$. 则:
$(axyzyxa)^0 = ((axyayzyayxa)^0 axyzxya(axyayzyayxa)^0)^0 = (axyayzyayxa)^0$

由定理 1知, $S \in QRBG$.

令 $e, f \in E(S)$, $a = (ef)^0$, $x = e$, $y = f$, $z = (ef)^0$. 则 S 满足的等式化为:

等式左边 $= (ef)^0ef(ef)^0fe(ef)^0 = ef$;

等式右边 $= ((ef)^0ef(ef)^0f(ef)^0f(ef)^0fe(ef)^0)^0(ef)^0ef(ef)^0ef(ef)^0((ef)^0ef(ef)^0f(ef)^0f(ef)^0fe(ef)^0)^0 = (ef)^0(ef)^0ef(ef)^0ef(ef)^0(ef)^0 = (ef)^2$.

所以新等式为: $ef = (ef)^2$,即 $(ef)^0 = ef$,由引理 2, $S \in O$.

令 $x, y \in S$, xHy, $a = x^0$, $z = (xy)^{-1}$,则 S 满足的等式化为:

等式左边 $= x^0xy(xy)^{-1}yxx^0 = x^0yxx^0 = yx$;

等式右边 $= (x^0xyx^0y(xy)^{-1}yx^0yxx^0)^0x^0xy(xy)^{-1}xyx^0(x^0xyx^0y(xy)^{-1}yx^0yxx^0)^0 = x^0xy(xy)^{-1}xyx^0 = xy$.

由此可得 $xy = yx$,说明在同一 H内满足交换性,则 $S \in HA$. 所以 $S \in O \cap QRBG \cap HA = OQRBA$.

另一方面,令 $S \in OQRBA$; $a, x, y, z \in S$.

由定理 1知, S 满足等式: $(axyzyxa)^0 = (axyayzyayxa)^0$. 则:

$axyzyxa = (axyzyxa)^0 axyzyxa(axyzyxa)^0 = (axyayzyayxa)^0 axyzyxa(axyayzyayxa)^0 =$
$(axyayzyayxa)^0 axyz(yx)^0y^0xyx^0(yx)^0a(axyayzyayxa)^0 =$
$(axyayzyayxa)^0 axyzxya(axyayzyayxa)^0$.

由此可得: $OQRBA \subseteq [axyzyxa = (axyayzyayxa)^0 axyzxya(axyayzyayxa)^0]$.

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Research into hotel room allocation based on EMSR

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Abstract: Currently, the room allocation of China's hotel management is at initial period based on experience. EMSR Model is used in seat optimization of aviation industry and good economic management suiting EMSR Model as relative stable production volume, non - storage of products, allocation of market, changeable market demand, cost structure with characteristics. By learning its application in aviation industry, EMSR Model was explored in hotel room optimization and allocation in order to provide reference for hotel room allocation decision.

Keywords: EMSR model; earning management; room allocation

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Several equations on quasiregular orthocryptogroup

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Abstract: In the studies on quasiregular orthocryptogroup, this paper gives the identity of quasiregular orthocryptogroup and the identity of quasiregular orthocryptogroup which is overabelian and so on.

Keywords: variety; quasiregular othocryptogroup; overabelian; orthodox

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