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具P-Laplacian的三阶边值问题正解的存在性

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摘 要:利用Guo-Krasnoselskii不动点定理,讨论了一类具P-Laplacian的边值问题正解的存在性,得到一些充分条件,扩充了以往文献的结果.

关键词:P-Laplacian; BVP; Guo-Krasnoselskii不动点定理
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考虑如下BVP:

$$(\varphi_p(u''))' + \lambda a(t)f(t,u(t),u'(t)) = 0, 0 < t < 1 \tag{1}$$

$$u(t) = \alpha u'(0), u(1) = \beta u(\eta), u''(0) = 0 \tag{2}$$

正解的存在性. 其中:

$$\varphi_p(s) = |s|^{p-2}s, p > 1, \varphi_q = \varphi_p^{-1}, \frac{1}{p} + \frac{1}{q} = 1, \alpha > 0, \eta \in (0,1), \beta\eta \in (0,1), \lambda > 0$$

具P-Laplacian算子的微分方程的边值问题在诸多领域如非牛顿力学、弹性理论等中有广泛的应用. 近年来,许多学者对此类BVP正解的存在性与多重性做了一系列的研究,并取得了一些成果,参见文献[1-6]. 文献[2]中,作者借助Guo-krasnoselskii不动点定理及Avery-Peterson不动点定理研究了BVP:

$$\begin{cases}(\varphi_p(u'))' + a(t)f(u,u') = 0, 0 < t < 1\\ u(0) = u'(0) = u(1) = 0\end{cases}$$

至少一个、两个或三个正解的存在性. 文献[3]与文献[4]中,作者利用Avery-Peterson不动点定理分别研究了:

$$(\varphi_p(u'))' + a(t)f(t,u(t),u'(t)) = 0, 0 < t < 1 \tag{3}$$

在多点边界条件:

$$u(0) = \sum_{i=1}^{n} \alpha_i u(\xi_i), u(1) = \sum_{i=1}^{n} \beta_i u(\xi_i); u(0) = \sum_{i=1}^{n-2} \alpha_i u_i(\xi_i), u'(1) = \sum_{i=1}^{n-2} \beta_i u'(\xi_i)$$

下多重正解的存在性. 文献[5]中,在方程(3)自治的情况下研究了其在边界条件:

$$\alpha\varphi_p(u(0)) - \beta\varphi_p(u'(\xi)) = 0, \gamma\varphi_p(u(1)) + \delta\varphi_p(u'(\eta)) = 0$$

下三个正解的存在性.

显然,上述文献及相关文献中讨论的往往是二阶情况,而对三阶的情况文献还比较少,尤其是对于方程(1)中允许 $a(t)$, $f(t,u,v)$ 在 $t=0, t=1$ 及 $u=0$ 处奇异的情况还未有涉及. 此处就是利用Guo-Krasnoselskii不动点定理讨论了式(1)(2)在上述情况下正解的存在性问题.

文中总假设以下条件成立:

(H_1) $\alpha > 0, \eta \in (0,1), \beta\eta \in (0,1), \Delta := (1-\beta\eta) + \alpha(1-\beta) > 0$.

(H_2) $a(t) \in C((0,1),(0,+\infty))$,允许 $a(t)$ 在 $t=0, t=1$ 处奇异,$a(t) \neq 0, t \in (0,1)$ 且 $0 < \int_0^1 a(\tau)\,d\tau < +\infty$.

(H_3) $f \in C([0,1] \times (0,+\infty) \times R, [0,+\infty))$,允许 $f(t,u,v)$ 在 $u=0$ 处奇异.

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1 预备知识

定义1 设E是一个实Banach空间,如果P是E中某个非空凸闭子集,并且满足下面两个条件:(1)若$x \in P, \lambda \geqslant 0$,则$\lambda x \in P$;(2)$x \in P, -x \in P$,则$x = 0$. 则称$P$是$E$中的一个锥.

定理1 设X是一个Banach空间,$P \subset X$是一个锥. 假设Ω_1, Ω_2是X中的两个有界开集,且$0 \in \Omega_1, \overline{\Omega_1} \subset \Omega_2$,$T: K \cap (\overline{\Omega_2} \backslash \Omega_1) \to K$是全连续算子,使得(i) $\|Tu\| \leqslant \|u\|, u \in K \cap \partial\Omega_1$,$\|Tu\| \geqslant \|u\|, u \in K \cap \partial\Omega_2$或(ii) $\|Tu\| \geqslant \|u\|, u \in K \cap \partial\Omega_1$,$\|Tu\| \leqslant \|u\|, u \in K \cap \partial\Omega_2$,则$T$在$K \cap (\overline{\Omega_2} \backslash \Omega_1)$中至少有一个不动点.

2 基本引理

令$E = C^1(0,1)$,定义范数$\|u\| = \max\limits_{0 \leqslant t \leqslant 1} |u(t)| + \max\limits_{0 \leqslant t \leqslant 1} |u'(t)|$,则$E$按上述范数构成Banach空间. 在$E$上定义锥$P = \{u(t): u(t) \geqslant 0, u(t)$在$(0,1)$上是凹的$\}$.

引理1 设$u \in P$,且满足式(2),则存在常数$\gamma > 0$,使得$\max\limits_{0 \leqslant t \leqslant 1} |u(t)| \leqslant \gamma \max\limits_{0 \leqslant t \leqslant 1} |u'(t)|$.

引理2 在条件(H_1)下,若$h(t) \in L^1[0,1]$,则BVP:

$$u''(t) + h(t) = 0, 0 < t < 1 \tag{4}$$

$$u(0) - \alpha u'(0) = 0, u(1) - \beta u(\eta) = 0 \tag{5}$$

有唯一解$u(t) = \int_0^1 G(t,s)h(s)\mathrm{d}s$. 其中:

$$G(t,s) = \begin{cases} \frac{1}{\Delta}(s+\alpha)((1-t)+\beta(t-\eta)), 0 \leqslant s \leqslant t \leqslant 1, 0 \leqslant s \leqslant \eta \leqslant 1 \\ \frac{1}{\Delta}((s+\alpha)(1-t)+\beta(t-s)(\eta+\beta)), 0 \leqslant \eta \leqslant s \leqslant t \leqslant 1 \\ \frac{1}{\Delta}(t+\alpha)((1-s)+\beta(s-\eta)), 0 \leqslant t \leqslant s \leqslant \eta < 1 \\ \frac{1}{\Delta}(t+\alpha)(1-s), 0 < \eta \leqslant t \leqslant s \leqslant 1 \end{cases}$$

注:令$P(t) = \min\left\{\frac{t+\beta}{1+\beta}, 1-t\right\}, q(s) = \frac{1}{\Delta}(s+\alpha)(1-s)$,则上述Green函数满足:

$$P(t)G(s,s) \leqslant G(t,s) \leqslant q(s) \tag{6}$$

易知$P(t)$为t的函数,$q(s)$为s的函数.

引理3 设条件$(H_1)-(H_3)$成立,则$u(t) \in C[0,1] \cap C^3(0,1)$是式(1)(2)的一个解,当且仅当$u(t) \in E$是积分方程:

$$u(t) = \int_0^1 G(t,s)\varphi_q\left(\int_0^s \lambda a(\tau) f(\tau, u(\tau), u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s \tag{7}$$

的一个解.

以上三个引理的证明从略.

对任意$u \in P$,定义算子$T: P \to E$

$$(Tu)(t) = \int_0^1 G(t,s)\varphi_q\left(\int_0^s \lambda a(\tau) f(\tau, u(\tau), u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s \tag{8}$$

引理4 设条件$(H_1)-(H_3)$满足,则$T: P \to P$是一个全连续算子.

证明 对$\forall u \in P$,由条件$(H_1)-(H_3)$及式(8)可知$Tu \in E$,$(Tu)(t) \geqslant 0, t \in [0,1]$,且经过计算可知$(Tu)''(t) \leqslant 0$,故$Tu$在区间$[0,1]$上是凹函数,故$T(P) \subset P$.

下证算子$T: P \to P$是一个全连续算子.

设D是P的任一有界集,则$\exists M > 0$,使得$D \subset \{u \in P: u \leqslant M\}$. 则可取$N = \max\limits_{(t,u,v) \in [0,1] \times [0,M] \times [-M,M]} f(t,$

$u,v)$,从而对 $\forall u\in D$,有:

$$|(Tu)(t)| = \left|\int_0^1 G(t,s)\varphi_q\left(\int_0^s \lambda a(\tau) f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right| \leqslant$$

$$\int_0^1 q(s)\varphi_q\left(\lambda N\int_0^s a(\tau)\mathrm{d}\tau\right)\mathrm{d}s \leqslant (\lambda N)^{\frac{1}{P-1}}\varphi_q\left(\int_0^1 a(\tau)\mathrm{d}\tau\right)\int_0^1 q(s)\mathrm{d}s =$$

$$\frac{1}{\Delta}\left(\alpha+\frac{1}{6}\right)(\lambda N)^{\frac{1}{P-1}}\varphi_q\left(\int_0^1 a(\tau)\mathrm{d}\tau\right)$$

当 $0<\eta\leqslant t\leqslant 1$ 时,

$$|(Tu)'(t)| = \left|\int_0^1 \frac{\mathrm{d}}{\mathrm{d}t}G(t,s)\varphi_q\left(\int_0^s \lambda a(\tau) f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right| =$$

$$\frac{1}{\Delta}\Big|\int_0^\eta (s+\alpha)(\beta-1)\varphi_q\left(\int_0^s \lambda a(\tau) f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s +$$

$$\int_\eta^t (\beta(\eta+\alpha)-(s+\alpha))\varphi_q\left(\int_0^s \lambda a(\tau) f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s +$$

$$\int_t^1 (1-s)\varphi_q\left(\int_0^s \lambda a(\tau) f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s\Big| \leqslant$$

$$\frac{(\lambda N)^{\frac{1}{P-1}}}{2\Delta}\varphi_q\left(\int_0^1 a(\tau)\mathrm{d}\tau\right)$$

当 $0\leqslant t\leqslant\eta\leqslant 1$ 时,

$$|(Tu)'(t)| = \left|\int_0^1 \frac{\mathrm{d}}{\mathrm{d}t}G(t,s)\varphi_q\left(\int_0^s \lambda a(\tau) f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right| =$$

$$\frac{1}{\Delta}\Big|\int_0^t (s+\alpha)(\beta-1)\varphi_q\left(\int_0^s \lambda a(\tau) f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s +$$

$$\int_t^\eta ((1-s)+\beta(s-\eta))\varphi_q\left(\int_0^s \lambda a(\tau) f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s +$$

$$\int_\eta^1 (1-s)\varphi_q\left(\int_0^s \lambda a(\tau) f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s\Big| \leqslant$$

$$\frac{1}{\Delta}\left(\frac{1}{2}+\beta\right)(\lambda N)^{\frac{1}{P-1}}\varphi_q\left(\int_0^1 a(\tau)\mathrm{d}\tau\right)$$

$$|(Tu)''(t)| = -\varphi_q\left(\int_0^s \lambda a(\tau) f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau\right) \leqslant (\lambda N)^{\frac{1}{P-1}}\varphi_q\left(\int_0^1 a(\tau)\mathrm{d}\tau\right)$$

可知 $T(P)$ 是等度连续的,从而由 Ascoli-Arzela 定理可知 $T(P)$ 为列紧的. 再由 Lebesgue 控制收敛定理知,T 是连续的. 因此 $T: P\to P$ 是全连续的.

证毕.

3 主要结论

记 $f_\infty = \lim\limits_{(|u|+|v|\to\infty)}\inf\min\limits_{0\leqslant t\leqslant 1}\dfrac{f(t,u,v)}{(|u|+|v|)^{P-1}}$,$f^0 = \lim\limits_{(|u|+|v|)\to 0}\sup\max\limits_{0\leqslant t\leqslant 1}\dfrac{f(t,u,v)}{(|u|+|v|)^{P-1}}$,$f_0 = \lim\limits_{(|u|+|v|)\to 0}\inf\min\limits_{0\leqslant t\leqslant 1}\dfrac{f(t,u,v)}{(|u|+|v|)^{P-1}}$,$f^\infty = \lim\limits_{(|u|+|v|)\to\infty}\sup\max\limits_{0\leqslant t\leqslant 1}\dfrac{f(t,u,v)}{(|u|+|v|)^{P-1}}$.

定理2 在条件$(\mathrm{H}_1)-(\mathrm{H}_3)$下,当 $f_\infty>0$,$f^0<\infty$ 时,如果 $\lambda\in(a,b)$,那么式(1)(2)至少存在一个正解,其中:

$$a = \frac{1}{\gamma^{P-1}\left(\int_0^1 P(t)G(s,s)\varphi_q\left(\int_0^s a(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right)^{P-1}f_\infty},\quad b = \frac{1}{\left(\int_0^1 q(s)\varphi_q\left(\int_0^s a(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right)^{P-1}f^0}$$

证明 (Ⅰ) 由 $\lambda\in(a,b)$ 可知,$\exists\varepsilon>0$,使得 $a_\varepsilon\leqslant\lambda\leqslant b_\varepsilon$,其中

$$a_\varepsilon = \frac{1}{\gamma^{P-1}\left(\int_0^1 P(t)G(s,s)\varphi_q\left(\int_0^s a(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right)^{P-1}(f_\infty - \varepsilon)},\quad b_\varepsilon = \frac{1}{\left(\int_0^1 q(s)\varphi_q\left(\int_0^s a(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right)^{P-1}(f^0 + \varepsilon)}$$

固定 ε,由 $f^0 < \infty$ 可知 $\exists M_1 > 0$,使得对于 $u: 0 < |u| + |u'| \leqslant M_1$,有:

$$f(t,u,u') \leqslant (f^0 + \varepsilon)(|u| + |u'|)^{P-1}$$

令 $\Omega_1 = \{u \in E: \|u\| < M_1\}$. 显然对 $\forall u \in P$,有 $\|Tu\| = (Tu)(t)$. 则对 $\forall u \in P \cap \partial\Omega_1$,有:

$$\begin{aligned}\|Tu\| = (Tu)(t) &= \int_0^1 G(t,s)\varphi_q\left(\int_0^s \lambda a(\tau)f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s \leqslant \\ &\int_0^1 q(s)\varphi_q\left(\int_0^s \lambda a(\tau)(f^0 + \varepsilon)(|u(\tau) + u'(\tau)|)^{P-1}\mathrm{d}\tau\right)\mathrm{d}s = \\ &(\lambda(f^0 + \varepsilon))^{\frac{1}{P-1}}(|u| + |u'|)\int_0^1 q(s)\varphi_q\left(\int_0^s a(\tau)\mathrm{d}\tau\right)\mathrm{d}s \leqslant \\ &(\lambda(f^0 + \varepsilon))^{\frac{1}{P-1}}\|u\|\int_0^1 q(s)\varphi_q\left(\int_0^s a(\tau)\mathrm{d}\tau\right)\mathrm{d}s\end{aligned}$$

(Ⅱ) 由 $f_\infty > 0$ 得,存在 $\hat{M}_2 > 0$,使得对 $\forall u: |u| + |u'| \geqslant \hat{M}_2$ 有 $f(t,u,u') \geqslant (f_\infty - \varepsilon)(|u| + |u'|)^{P-1}$.

取 $M_2 = \max\{\gamma M_1, \gamma\hat{M}_2\}$,令 $\Omega_2 = \{u \in E: \|u\| < M_2\}$,对 $\forall u \in P \cap \partial\Omega_2$,由引理1知:$\gamma\|u\| \leqslant \|u\| + \|u'\| \leqslant \|u\|$,$t \in [0,1]$,故:

$$\begin{aligned}\|Tu\| = (Tu)(t) &= \int_0^1 G(t,s)\varphi_q\left(\int_0^s \lambda a(\tau)f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s \geqslant \\ &\int_0^1 P(t)G(s,s)\varphi_q\left(\int_0^s \lambda a(\tau)(f_\infty - \varepsilon)(|u(\tau)| + |u'(\tau)|)^{P-1}\mathrm{d}\tau\right)\mathrm{d}s \geqslant \\ &(\lambda(f_\infty - \varepsilon))^{\frac{1}{P-1}}\gamma\|u\|\int_0^1 P(t)G(s,s)\varphi_q\left(\int_0^s a(\tau)\mathrm{d}\tau\right)\mathrm{d}s \geqslant \|u\|\end{aligned}$$

从而根据Guo-Krasnoselskii不动点定理,T 至少存在一个不动点 $u^* \in P \cap (\overline{\Omega_2} \setminus \Omega_1)$,且据 $(Tu^*)''(t) \leqslant 0$ 及定义的 P, Ω_1, Ω_2,可知 $u^*(t)$ 是式(1)(2)的一个正解. 证毕

定理3 在条件(H_1)-(H_3)下,当 $f_0 > 0$,$f^\infty < \infty$ 时,如果 $\lambda \in (c,d)$,那么式(1)(2)至少存在一个正解,其中:

$$c = \frac{1}{\gamma^{P-1}\left(\int_0^1 P(t)G(s,s)\varphi_q\left(\int_0^s a(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right)^{P-1}f_0},\quad d = \frac{1}{2^{P-1}\left(\int_0^1 q(s)\varphi_q\left(\int_0^s a(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right)^{P-1}f^\infty}$$

证明 (Ⅰ) 由 $\lambda \in (c,d)$ 可得 $\exists \varepsilon > 0$,使得 $c_\varepsilon \leqslant \lambda \leqslant d_\varepsilon$,其中:

$$c_\varepsilon = \frac{1}{\gamma^{P-1}\left(\int_0^1 P(t)G(s,s)\varphi_q\left(\int_0^s a(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right)^{P-1}(f_0 - \varepsilon)}$$

$$d_\varepsilon = \frac{1}{2^{P-1}\left(\int_0^1 q(s)\varphi_q\left(\int_0^s a(\tau)\mathrm{d}\tau\right)\mathrm{d}s\right)^{P-1}(f^\infty + \varepsilon)}$$

固定 ε,由 $f_0 > 0$ 可知 $\exists M_1 > 0$,使得对 $\forall u: 0 \leqslant |u| + |u'| \leqslant M_1$,$f(t,u,u') \geqslant (f_0 - \varepsilon)(|u| + |u'|)^{P-1}$,令 $\Omega_1 = \{u \in E: \|u\| < M_1\}$,显然 $\forall u \in P \cap \partial\Omega_1$,$\|Tu\| = (Tu)(t)$,由引理1知对 $\forall u \in P \cap \partial\Omega_1$ 时,有:

$$\gamma\|u\| \leqslant |u| + |u'| \leqslant \|u\|,\ t \in [0,1]$$

$$\begin{aligned}\|Tu\| = (Tu)(t) &= \int_0^1 G(t,s)\varphi_q\left(\int_0^s \lambda a(\tau)f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau\right)\mathrm{d}s \geqslant \\ &\int_0^1 P(t)G(s,s)\varphi_q\left(\int_0^s \lambda a(\tau)(f_0 - \varepsilon)(|u(\tau)| + |u'(\tau)|)^{P-1}\mathrm{d}\tau\right)\mathrm{d}s \geqslant \\ &(\lambda(f_0 - \varepsilon))^{\frac{1}{P-1}}\gamma\|u\|\int_0^1 P(t)G(s,s)\varphi_q\left(\int_0^s a(\tau)\mathrm{d}\tau\right)\mathrm{d}s \geqslant \|u\|\end{aligned}$$

(Ⅱ) 由 $f^\infty < \infty$ 可知 $\exists \hat{M}_2 > 0$,使 $|u| + |u'| \geqslant \hat{M}_2$ 有:$f(t,u,u') \leqslant (f^\infty + \varepsilon)(|u| + |u'|)^{P-1}$. 分两种情况讨论:

情况 1　f 有界，即 $\exists N > 0$，使得 $f(t,u,u') \leqslant N$，对 $\forall t \in [0,1]$，$0 \leqslant u \leqslant +\infty$，$u' \in R$ 成立. 取 $M_2 = \max\{\gamma M_1, (\lambda N)^{\frac{1}{P-1}} \int_0^1 q(s)\varphi_q(\int_0^s a(\tau)\mathrm{d}\tau)\mathrm{d}s\}$，且令 $\Omega_2 = \{u \in E: \|u\| < M_2\}$，当 $u \in P \cap \partial\Omega_2$ 时，有:

$$\|Tu\| = (Tu)(t) = \int_0^1 G(t,s)\varphi_q(\lambda a(\tau)f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau)\mathrm{d}s \leqslant$$

$$(\lambda N)^{\frac{1}{P-1}} \int_0^1 q(s)\varphi_q(\int_0^s a(\tau)\mathrm{d}\tau)\mathrm{d}s \leqslant M_2 = \|u\|$$

情况 2　f 无界，易知 $\exists M_2 > \max\left\{M_1, \frac{\hat{M}_2}{\gamma}\right\}$，使得 $f(t,u,u') \leqslant f(t,M_2,M_2)$ 对 $\forall t \in [0,1]$，$0 \leqslant |u| + |u'| \leqslant M_2$，取 $\Omega_2 = \{u \in E: \|u\| < M_2\}$，对 $\forall u \in P \cap \partial\Omega_2$，有:

$$\|Tu\| = (Tu)(t) = \int_0^1 G(t,s)\varphi_q(\int_0^s \lambda a(\tau)f(\tau,u(\tau),u'(\tau))\mathrm{d}\tau)\mathrm{d}s \leqslant$$

$$\int_0^1 q(s)\varphi_q(\int_0^s \lambda a(\tau)f(\tau,M_2,M_2)\mathrm{d}\tau)\mathrm{d}s \leqslant$$

$$(\lambda(f^\infty + \varepsilon))^{\frac{1}{P-1}} 2M_2 \int_0^1 q(s)\varphi_q(\int_0^s a(\tau)\mathrm{d}\tau)\mathrm{d}s \leqslant M_2 = \|u\|$$

从而根据 Guo-Krasnoselskii 不动点定理，T 至少存在一个不动点 $u^* \in P \cap (\bar{\Omega}_2 \backslash \Omega_1)$，且据 $(Tu^*)''(t) \leqslant 0$ 及定义的 P, Ω_1, Ω_2 可知，$u^*(t)$ 是式(1)(2)的一个正解. 证毕

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Existence of Positive Solution of Third-order Boundary Value Problem with P-Laplacian

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Abstract: In this paper, by using Guo-Krasnoselskii fixed point theorem, we discuss the existence of positive solutions for a class of boundary value problem with P-Laplacian and obtain some sufficient conditions which expand the results of previous works.

Key words: P-Laplacian; BVP; Guo-Krasnoselskii fixed point theorem

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