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一类 B预不变凸多目标规划的最优性充分条件

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摘　要:在 B预不变广义凸性条件下 ,研究一类多目标规划问题 ,得到了 Kuhn-Tucker型最优性充分条件.

关键词:多目标规划 ;最优性条件 ; B预不变凸函数 ;有效解 ;弱有效解

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设 E^n 为 n维欧氏空间 ,以下引入几个向量不等式记号: $y=(y_1, y_2, \cdots, y_n)^T$, $z=(z_1, z_2, \cdots, z_n)^T\in E^n$,并规定 $y=z$的充要条件是: $y_i=z_i$, $i=1,2,\cdots,n$; $y>z$的充要条件是: $y_i>z_i$, $i=1,2,\cdots,n$; $y\geqq z$的充要条件是: $y_i\geqq z_i$, $i=1,2,\cdots,n$; $y\geq z$的充要条件是: $y_i\geqq z_i$, $i=1,2,\cdots,n$,但至少存在一个 $1\leqq j_0\leqq n$,使 $y_{j_0}>z_{j_0}$,即 $y\neq z$ 同样 ,可以定义 $y<z$, $y\leqq z$和 $y\leq z$

考虑下面的多目标规划问题:

$$(\mathrm{VP})\quad \min f(x)\qquad \mathrm{s\ t}\ g_j(x)\leqq 0,\ h_k(x)=0$$

其中 $f_i, g_j, h_k: E^n\to E^1$; $i=1,\cdots,p$; $j=1,\cdots,m$; $k=1,\cdots,s$; $f(x)=(f_1(x), f_2(x),\cdots,f_p(x))^T$; $g(x)=(g_1(x), g_2(x),\cdots,g_m(x))^T$; $h(x)=(h_1(x),h_2(x),\cdots,h_s(x))^T$; $R=\{x\in E^n: g(x)\leqq 0,\ h(x)=0\}$为 (VP)的可行集.

将对此多目标规划问题中目标函数和约束函数进行 B预不变广义凸性假定 ,研究它的最优性充分条件 ,为此引入相关定义:

定义 1　设 $\bar{x}\in R$,如果不存在 $x\in R$,使 $f(x)\leq f(\bar{x})$ (或 $f(x)<f(\bar{x})$),则说 $\bar{x}$是 (VP)的有效解 (或弱有效解).

定义 2[1]　称 $X\subseteq E^n$ 为不变凸集 ,若存在向量值函数 $\eta(x,y)$,使得 $\forall x, y\in X$, $\theta\in[0,1]$, $y+\theta\eta(x,y)\in X$.

定义 3[2]　设 $X\subseteq E^n$ 为不变凸集 ,称 $f(x)$关于相同的 $\eta(x,y)$为预不变凸函数 ,若 $\forall x, y\in X$, $\theta\in[0,1]$, $f(y+\theta\eta(x,y))\leqq\theta f(x)+(1-\theta)f(y)$.

定义 4[3]　设 $X\subseteq E^n$ 为非空集合 , $b: X\times X\times[0,1]\to E_+$,称 $f(x)$在 y处关于 b为 B - 凸函数 ,若 $\forall x\in X$, $\theta\in[0,1]$,有 $1-\theta b(x,y,\theta)\geqq 0$,且 $f(y+\theta(x-y))\leqq\theta b(x,y,\theta)f(x)+(1-\theta b(x,y,\theta))f(y)$.

定义 5[4]　设 $X\subseteq E^n$ 为不变凸集 , $f: X\to E^1$ 可微 ,又设 $\bar{b}: X\times X\to E_+$,其中 $\bar{b}(x,y)=\lim\limits_{\theta\to 0^+}b(x,y,\theta)\geqq 0$,称 $f(x)$在 y处关于 $\bar{b}$和 $\eta(x,y)$为 B - 不变凸函数 ,若 $\bar{b}(x,y)[f(x)-f(y)]\geqq\eta(x,y)^T\nabla f(y)$, $\forall x\in X$.

定义 6[5]　设 $X\subseteq E^n$ 为不变凸集 , $f: X\to E^1$ 可微 ,又设 $b: X\times X\times[0,1]\to E_+$,称 $f(x)$在 y处关于 b和 $\eta(x,y)$为 B - 预不变凸函数 ,若 $\forall x\in X$, $\theta\in[0,1]$,有 $1-\theta b(x,y,\theta)\geqq 0$,且 $f(y+\theta\eta(x,y))\leqq\theta b(x,y,\theta)f(x)+(1-\theta b(x,y,\theta))f(y)$.

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1　预备知识

这一部分主要介绍一些在证明中会用到的引理.

引理 1[6]　设 $\bar{\lambda}\in\Lambda^{+}(\Lambda^{++})$,则单目标:

$$(SP)_{\bar{\lambda}}\ \min_{x\in R}\bar{\lambda}^{T}f(x)$$

的最优解 $\bar{x}$是(VP)的弱有效解(有效解),其中,$\Lambda^{+}=\{\lambda=(\lambda_1,\lambda_2,\cdots,\lambda_p)^{T}\mid$ 诸 $\lambda_i\geqq 0$ 且 $\sum_{i=1}^{p}\lambda_i=1\}$,

$\Lambda^{++}=\{\lambda=(\lambda_1,\lambda_2,\cdots,\lambda_p)^{T}\mid$ 诸 $\lambda_i>0$ 且 $\sum_{i=1}^{p}\lambda_i=1\}$.

引理 2[7]　若 $f(x)$为关于 b和 η的 B-预不变凸函数,则存在 $\bar{b}: X\times X\to E_{+}$,使得 $\bar{b}(x,y)[f(x)-f(y)]\geqq\eta(x,y)^{T}\nabla f(y);\ \forall x,y\in X$. 其中 $\bar{b}(x,y)=\lim_{\theta\to 0^{+}}b(x,y,\theta)\geqq 0$.

2　主要结论

定理 1　设 $X\subseteq E^{n}$ 关于 η为开不变凸集,$f(x)$, $g(x)$和 $h(x)$均连续可微; $\bar{x}\in R$, $f_i(x)$ $(i=1,2,\cdots,p)$, $g_j(x)$ $(j\in J(\bar{x}))$, $\pm h_k(x)$ $(k=1,2,\cdots,s)$都是在点 $\bar{x}$关于 b和 $\eta(x,\bar{x})$的 B-预不变凸函数;对于 $\bar{\lambda}\geq 0$ $(\bar{\lambda}>0)$,如果存在 $\bar{u}\in E^{m}$, $\bar{v}\in E^{s}$ 使得 $(\bar{x},\bar{\lambda},\bar{u},\bar{v})$满足 Kuhn-Tucker条件:

$$\bar{\lambda}^{T}\nabla f(\bar{x})+\bar{u}^{T}\nabla g(\bar{x})+\bar{v}^{T}\nabla h(\bar{x})=0 \tag{Ⅰ}$$

$$\bar{u}^{T}g(\bar{x})=0,\ \bar{u}\geqq 0 \tag{Ⅱ}$$

则 $\bar{x}$是(VP)的弱有效解(有效解).

其中 $J(\bar{x})=\{j\mid g_j(\bar{x})=0,1\leqq j\leqq m\}$; $\bar{\lambda}=(\bar{\lambda}_1,\bar{\lambda}_2,\cdots,\bar{\lambda}_p)^{T}$; $\bar{u}=(\bar{u}_1,\bar{u}_2,\cdots,\bar{u}_m)^{T}$, $\bar{v}=(\bar{v}_1,\bar{v}_2,\cdots,\bar{v}_s)^{T}$.

$$\nabla f(x)=\begin{pmatrix}\frac{\partial f_1(x)}{\partial x_1} & \cdots & \frac{\partial f_1(x)}{\partial x_n}\\ \cdots & \ddots & \cdots\\ \frac{\partial f_p(x)}{\partial x_1} & \cdots & \frac{\partial f_p(x)}{\partial x_n}\end{pmatrix}$$

$$\nabla g(x)=\begin{pmatrix}\frac{\partial g_1(x)}{\partial x_1} & \cdots & \frac{\partial g_1(x)}{\partial x_n}\\ \cdots & \ddots & \cdots\\ \frac{\partial g_m(x)}{\partial x_1} & \cdots & \frac{\partial g_m(x)}{\partial x_n}\end{pmatrix}$$

$$\nabla h(x)=\begin{pmatrix}\frac{\partial h_1(x)}{\partial x_1} & \cdots & \frac{\partial h_1(x)}{\partial x_n}\\ \cdots & \ddots & \cdots\\ \frac{\partial h_k(x)}{\partial x_1} & \cdots & \frac{\partial h_k(x)}{\partial x_n}\end{pmatrix}$$

证明　对给定的(VP)和 $\bar{\lambda}$,作单目标规划:

$$(SP)_{\bar{\lambda}}\ \min_{x\in R}\bar{\lambda}^{T}f(x)$$

注意到 $\bar{\lambda}\geq 0$ $(\bar{\lambda}>0)$,因此不妨设 $\bar{\lambda}\in\Lambda^{+}(\Lambda^{++})$,于是$(SP)_{\bar{\lambda}}$便可视为引理 1问题,从而只需要证明 $\bar{x}$是$(SP)_{\bar{\lambda}}$的最优解即可.

设 $x\in R$, $g_j(x)\leqq g_j(\bar{x})=0$, $j\in J(\bar{x})$. 由于 $g_j(x)$是在点 $\bar{x}$关于 b和 $\eta(x,\bar{x})$的 B-预不变凸函数,根据引理 2有 $\bar{b}(x,\bar{x})[g_j(x)-g_j(\bar{x})]\geqq\eta(x,\bar{x})^{T}\nabla g_j(\bar{x})$,其中 $\bar{b}(x,\bar{x})=\lim_{\theta\to 0^{+}}b(x,\bar{x},\theta)\geqq 0$,所以 $\eta(x,\bar{x})^{T}\nabla g_j(\bar{x})\leqq$

0,从而 $\eta(x,\bar{x})^{\mathrm{T}}\nabla g_j(\bar{x})\bar{u}_j\leqq 0,\ j\in J(\bar{x})$.

当 $j\notin J(\bar{x})$时, $g_j(\bar{x})<0$,从而根据式(Ⅱ),有 $\bar{u}_j=0$,故:

$$\eta(x,\bar{x})^{\mathrm{T}}\nabla g_j(\bar{x})\bar{u}_j\leqq 0;\ j=1,\cdots,m \tag{1}$$

又因为 $\pm h_k(x)$是在点 $\bar{x}$关于 b和 $\eta(x,\bar{x})$的 B-预不变凸函数, $\forall x\in R$,根据引理 2有:

$$\bar{b}(x,\bar{x})[h_k(x)-h_k(\bar{x})]\geqq\eta(x,\bar{x})^{\mathrm{T}}\nabla h_k(\bar{x});\ k=1,2,\cdots,s$$

$$\bar{b}(x,\bar{x})[h_k(x)-h_k(\bar{x})]\leqq\eta(x,\bar{x})^{\mathrm{T}}\nabla h_k(\bar{x});\ k=1,2,\cdots,s$$

其中, $\bar{b}(x,\bar{x})=\lim\limits_{\theta\to0^+}b(x,\bar{x},\theta)\geqq0$. 所以 $\bar{b}(x,\bar{x})[h_k(x)-h_k(\bar{x})]=\eta(x,\bar{x})^{\mathrm{T}}\nabla h_k(\bar{x})=0;\ k=1,2,\cdots,s$,即:

$$(x,\bar{x})^{\mathrm{T}}\nabla h_k(\bar{x})\bar{v}_k=0;\ k=1,2,\cdots,s \tag{2}$$

由式(1)(2)可得 $\eta(x,\bar{x})^{\mathrm{T}}\sum\limits_{j=1}^{m}\nabla g_j(\bar{x})\bar{u}_j+\eta(x,\bar{x})^{\mathrm{T}}\sum\limits_{k=1}^{s}\nabla h_k(\bar{x})\bar{v}_k\leqq0$两端转置,有 $\sum\limits_{j=1}^{m}\bar{u}_j\nabla g_j(\bar{x})^{\mathrm{T}}\eta(x,\bar{x})+\sum\limits_{k=1}^{s}\bar{v}_k\nabla h_k(\bar{x})^{\mathrm{T}}\eta(x,\bar{x})\leqq0$,即:

$$[\bar{u}^{\mathrm{T}}\nabla g(\bar{x})+\bar{v}^{\mathrm{T}}\nabla h(\bar{x})]\eta(x,\bar{x})\leqq0 \tag{3}$$

式(1)两端同时右乘 $\eta(x,\bar{x})$,得:

$$\bar{\lambda}^{\mathrm{T}}\nabla f(\bar{x})\eta(x,\bar{x})+\bar{u}^{\mathrm{T}}\nabla g(\bar{x})\eta(x,\bar{x})+\bar{v}^{\mathrm{T}}\nabla h(\bar{x})\eta(x,\bar{x})=0 \tag{4}$$

由式(3)(4)可得 $\bar{\lambda}^{\mathrm{T}}\nabla f(\bar{x})\eta(x,\bar{x})\geqq0$,而:

$$\bar{\lambda}^{\mathrm{T}}\nabla f(\bar{x})\eta(x,\bar{x})=\sum_{i=1}^{p}\bar{\lambda}_i\nabla f_i(\bar{x})^{\mathrm{T}}\eta(x,\bar{x}) \tag{5}$$

对式(5)右端进行转置,可得:

$$\eta(x,\bar{x})^{\mathrm{T}}\sum_{i=1}^{p}\nabla f_i(\bar{x})\bar{\lambda}_i=\eta(x,\bar{x})^{\mathrm{T}}\nabla f_1(\bar{x})\bar{\lambda}_1+\eta(x,\bar{x})^{\mathrm{T}}\nabla f_2(\bar{x})\bar{\lambda}_2+\cdots+\eta(x,\bar{x})^{\mathrm{T}}\nabla f_p(\bar{x})\bar{\lambda}_p\geqq0$$

因为 $f_i(x)\ (i=1,2,\cdots,p)$都是在点 $\bar{x}$关于 $\bar{b}$和 $\eta(x,\bar{x})$的 B-预不变凸函数,根据引理 2,有:

$$\bar{b}(x,\bar{x})[f_1(x)-f_1(\bar{x})]\bar{\lambda}_1+\bar{b}(x,\bar{x})[f_2(x)-f_2(\bar{x})]\bar{\lambda}_2+\cdots+\bar{b}(x,\bar{x})[f_p(x)-f_p(\bar{x})]\bar{\lambda}_p=$$

$$\bar{b}(x,\bar{x})\{[f_1(x)\bar{\lambda}_1+f_2(x)\bar{\lambda}_2+\cdots+f_p(x)\bar{\lambda}_p]-[f_1(\bar{x})\bar{\lambda}_1+f_2(\bar{x})\bar{\lambda}_2+\cdots+f_p(\bar{x})\bar{\lambda}_p]\}=$$

$$\bar{b}(x,\bar{x})[\sum_{i=1}^{p}f_i(x)\bar{\lambda}_i-\sum_{i=1}^{p}f_i(\bar{x})\bar{\lambda}_i]\geqq$$

$$\eta(x,\bar{x})^{\mathrm{T}}\nabla f_1(\bar{x})\bar{\lambda}_1+\eta(x,\bar{x})^{\mathrm{T}}\nabla f_2(\bar{x})\bar{\lambda}_2+\cdots+\eta(x,\bar{x})^{\mathrm{T}}\nabla f_p(\bar{x})\bar{\lambda}_p\geqq0$$

其中 $\bar{b}(x,\bar{x})=\lim\limits_{\theta\to0^+}b(x,\bar{x},\theta)\geqq0$. 即 $\bar{b}(x,\bar{x})[\sum\limits_{i=1}^{p}f_i(x)\bar{\lambda}_i-\sum\limits_{i=1}^{p}f_i(\bar{x})\bar{\lambda}_i]\geqq0$. 所以 $\sum\limits_{i=1}^{p}f_i(x)\bar{\lambda}_i-\sum\limits_{i=1}^{p}f_i(\bar{x})\bar{\lambda}_i\geqq0$, 即 $\bar{\lambda}^{\mathrm{T}}f(x)-\bar{\lambda}^{\mathrm{T}}f(\bar{x})\geqq0$,从而 $\bar{\lambda}^{\mathrm{T}}f(x)\geqq\bar{\lambda}^{\mathrm{T}}f(\bar{x})$. 故 $\bar{x}$是 $(\mathrm{SP})_{\bar{\lambda}}$ 的最优解,亦是(VP)的弱有效解(有效解).

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Optimal sufficient conditions of a class of multiobjective programming about B-preinvex function

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Abstract: Under the assumption of B-preinvex function, this paper studies a class of multi-objective programming problems, and obtains Kuhn-Tunker sufficient optimality conditions

Key words: multiobject programming; optimal conditions; B-preinvex function; efficient solution; weakly efficient solution

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Minimax estimation in multivariate linear regression model under restrictions

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Abstract: In this paper, a multivariate Minimax estimation in all linear estimators under the loss function $\mathrm{tr}(\hat{B} - B)' A(\hat{B} - B)$ is derived. We consider its properties In special case, the multivariate Minimax estimation includes power Ridge regression, Stein 's estimator and so on.

Key words: multivariate linear regression model; multivariate linear Minimax estimation; multivariate Ridge regression estimation

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